CHAPTER 4

# Connections

"Given a pile of jigsaw puzzle pieces and told to put them together, no doubt we would ask to see the picture they make . . . Without the picture, we probably wouldn't want to bother with the puzzle. Ironically, this situation is very much like what we ask young people to do all the time in school. To students, the typical curriculum presents an endless array of facts and skills that are unconnected, fragmented, and disjointed…"

Beane, 1991

# Chapter 4

## Introduction: The Importance of Connections

Traditionally, the American mathematics curriculum has been organized around specific content areas. Defining mathematics in terms of arithmetic, algebra, geometry, or any mathematical content area, is like trying to define an elephant by describing a few of its parts. To define mathematics in this way ignores the rich connections between the content areas and, more importantly, the fundamental act of *doing mathematics.* Mathematics is the language and science of patterns which involves conjecturing, testing, modeling, identifying patterns, verifying, analyzing, and making generalizations. These processes can be developed more fully if they are experienced throughout the various content strands of mathematics. The interrelationships between and among concepts need to be brought to students' conscious levels in ways that help them see the connected nature of mathematics and its usefulness in other disciplines and real-life experiences.

*"Among the various aims we consider important in education, two are especially so. We would like our children to be well informed — that is, to understand ideas that are important, useful, beautiful, and powerful. And we also want them to have the appetite and ability to think analytically and critically, to be able to speculate and imagine, to see connections among ideas and to be able to use what they know to enhance their own lives and to contribute to their culture."*
Eisner (1997)

Making connections is an important human activity. In their study of the workings of the brain, Caine and Caine (1990, pp. 66-69) document that seeking patterns and connections is the natural activity of the brain. Isolated pieces of information require more time to assimilate than learning experiences that are connected with a person's prior knowledge. Students come to school with a variety of experiences and personal constructs about mathematics. When students learn a new concept they naturally make connections to what they already know and develop their own understandings in ways that make sense to them. Students may make accurate connections to previous knowledge, or they may develop or reinforce misconceptions that are very difficult to change. It is the role of the teacher to help students build accurate constructs about mathematical ideas and be able to apply their knowledge to both predictable and unpredictable situations. When teachers take account of students' prior knowledge and are familiar with students' cultural backgrounds, learning style preferences, and experiences in and out of school, they increase the opportunities to foster important connections and facilitate student understanding of mathematics at higher levels. (See also, *Chapter 2* of this *Framework.*)

It is impossible to teach students all the mathematical content that does (or will) exist or cover all the applications they will need to successfully navigate in our technological world and knowledge-based society. The body of mathematical knowledge and applications is immense and growing at an exponential rate. Educators are now faced with identifying and focusing on the key content that should be taught in mathematics. The content strands developed in Chapter 3 of this *Framework* and emphasized in the *Minnesota Graduation Standards* are a guide to help teachers and districts make those decisions. A strong, interconnected foundation of mathematical knowledge is important to develop students' mathematical literacy and empower them to learn new mathematics and apply it to everyday situations and throughout their careers.

The release of the Third International Mathematics and Science Study (TIMSS) results for eighth grade serves to focus discussion on what exemplifies a well designed mathematics curriculum. A sound K-12 curriculum should be focused and coherent. The key ideas should be based on rich tasks that challenge students' thinking and connect students' school experiences to previous and future learning. Unfortunately, the authors of the TIMSS analysis have characterized the curricula in American schools *as a mile wide and an inch deep*, indicating the rapid and shallow study of many topics in mathematics throughout each year of schooling (NCES, 1996; Schmidt, McKnight & Raizen, 1997). This pattern reduces the opportunities available to engage students in the focused study of any given topic and leaves little time for deep understanding to evolve. Without this depth and application of knowledge, it will be hard for students to compete mathematically in the global economy in which they will live and work.

*"...mathematical power is a function of a students' prior knowledge and experience and the ability to connect that knowledge in productive ways to new contexts."*
NAGB, 1996

A mathematically literate person must not only have an understanding of mathematical content, but also be able to make connections among mathematical topics and to apply mathematical knowledge and understandings to other disciplines and to a variety of life situations. The National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989), the *Minnesota Graduation Standards,* and the *Minnesota K-12 Mathematics Framework* all describe a vision of mathematics education that addresses this view of mathematical knowledge. The performance assessment process in the *Minnesota Graduation Standards* reinforces the need for connections between schooling and daily life experiences by outlining a performance assessment process that will validate achievement throughout a student's K-12 education. When implemented, these standards and assessments will result in a collection of coherent experiences that students can use in and out of school as a basis for their citizenship in a world that increasingly relies on mathematics, science, and technology.

## Types of Connections

The connections described in this *Framework* are organized into three parts:

- interrelationships within mathematics
- connections between mathematics and other disciplines
- application-based connections

This classification system does not exhaust all the possibilities, but it is helpful in thinking about how mathematics might be organized and taught. This chapter on *Connections* is intended to provide a starting point for teachers in structuring curriculum, instruction, and assessment to help students make the connections that are at the heart of national and state standards. Connections among the various strands of mathematics, connections of key mathematical concepts to other disciplines, and connections to real life experiences, including applications in the workplace, are described. In addition, tools and models for planning curriculum connections are provided throughout this chapter. The following is an outline of this chapter.

*"...there is widespread consensus that mathematics...must be presented as a connected discipline rather than a set of discrete topics, and that it must be learned in meaningful contexts that connect mathematics to other subjects, to the interests and experiences of students."*
House, 1995

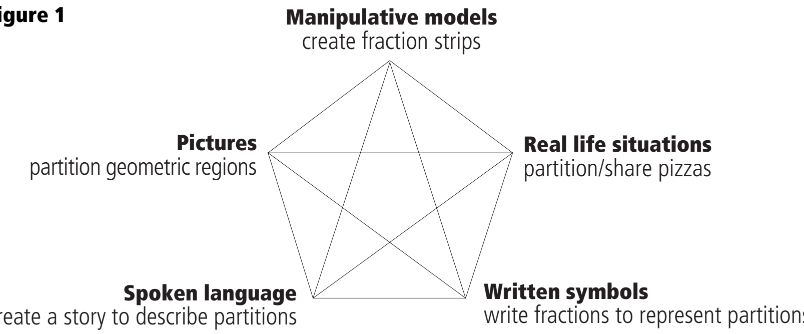
## Part 1: Interrelationships within Mathematics

### Connections among Representations of a Concept

*"...a mathematics program that does not make use of the environment to develop mathematical concepts eliminates the first and perhaps the most crucial... representation of an idea."*
Post, 1992

The research of Piaget, Dienes, and Bruner suggests that children's concepts evolve from direct interaction with the environment. Clements and McMillen (1996) capture this when they write, "Mathematical ideas are ultimately...integrated...not by their physical or real-world characteristics but rather by how 'meaningful'—connected to other ideas and situations—they are" (p. 271). Lesh (1979) found that using manipulatives in conjunction with pictorial, verbal, symbolic, and real-world representations can maximize learning. Connections to, and translations between, different representations of a concept are important cognitive processes which lead to a more robust understanding of concepts. The example shown in Figure 1 applies the Lesh model to various representations of fractions.

**Figure 1**




### Connections within or across Mathematical Strands

Without an explicit focus on connections, students may view their learning of mathematics as the accumulation of unrelated and discrete ideas, and have little understanding of how mathematical ideas are related. Connections that draw together key ideas and topics within or across mathematical strands help students develop a more coherent understanding of the concept or process they are learning. (See examples in Table 1.)

**Table 1 – Interrelationships within Mathematics**

| | Examples of connections among topics WITHIN a content strand | Examples of connections ACROSS mathematical strands |
|---|---|---|
| **Primary** | view plane shapes as the "footprints" of solid shapes | relate numerical patterns to geometric patterns |
| **Intermediate** | investigate alternate algorithms that show the relationship of division to subtraction | collect data from circular objects to investigate the value of $\pi$ |
| **Middle School** | recognize and flexibly interchange equivalent fractions, decimals, and percents | use algebraic expressions to communicate the effects of scaling on area and perimeter |
| **High School** | use a variety of counting techniques including permutations, combinations, tree diagrams, and matrices | use both algebraic and geometric representations to describe transformations |

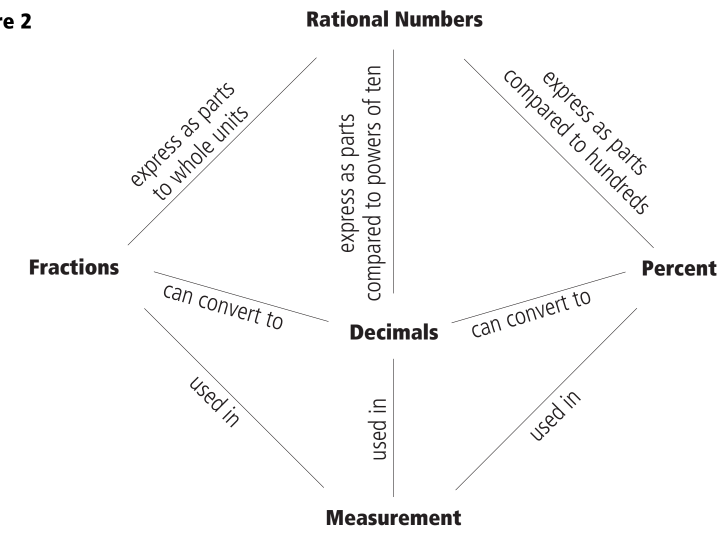
*"By examining many different strands of mathematics, we gain perspective on common features and dominant ideas...To grow mathematically, children must be exposed to a rich variety of patterns appropriate to their own lives through which they can see variety, regularity, and interconnections."*
Steen, 1990

Teachers can support the development of these connections through careful unit construction, appropriate questioning, and by using and teaching students to use devices such as graphic organizers (e.g. concept maps, mind maps, webs, backmaps, sorting and classification trees). The intent is to help students gain conceptual understanding that stretches beyond the bounds of one unit of instruction or one grade level. New curriculum projects that integrate mathematical strands and articulate concept development across the K-12 spectrum support both teachers and students in identifying and understanding interrelationships within mathematics and seeing it as a unified discipline. Assessment then focuses on students' understanding of major concepts, rather than on disconnected elements of mathematics.

In planning a unit of study, even one that closely follows a textbook, teachers must clearly understand the interrelationships of the various ideas within the unit if they are to help students see and appreciate these connections. A teacher can use a concept map to plan a unit of study. In essence, a concept map creates a "roadmap" for planning pathways through the unit. It is also important that teachers reflect on other concepts the students have previously studied so that new ideas can be well-grounded in past experiences.

For example, many students view fractions, decimals, and percents as three isolated topics, connected only because they are asked at some point to convert among them. If they learn about these concepts as different but equivalent representations of rational numbers, they have a better chance of understanding why and when, as well as how, to convert among the representations. The concept map in Figure 2 shows some connections that should be emphasized in a unit of study.

**Figure 2**




*"When faced with thematic challenges... students call on their available mathematical knowledge and problem-solving expertise. They do not ask 'Are we doing computation, geometry, or statistics today?' but rather, 'What do we need to figure out? What do we know that might help? And how might we go about it?'"*
Kleiman, 1995

### Integrated Mathematics Curriculum

In the *Curriculum and Evaluation Standards for School Mathematics* (1989), NCTM calls for a core curriculum at the secondary level that would allow all students to integrate major concepts of geometry and algebra along with concepts from statistics, probability, data analysis, and discrete mathematics. The National Science Foundation (NSF) has sponsored the development of curricula that focus instruction on key mathematical concepts across the various content strands in mathematics. A characteristic of these curricula is the embedding of key concepts within a series of problem investigations in a connected unit of study. As students investigate a real-life problem or a set of problems, they draw upon various areas of mathematics, deepening their understanding of important ideas. While each unit has a specific mathematical focus, most units are structured around a central problem that brings in other topics as needed, rather than narrowly and artificially restricting the mathematical content. Ideas that are developed in one unit are revisited and deepened in later units.

An important goal of these NSF funded curricula is to make a core mathematics curriculum accessible to *all* students. Toward that end, these programs have been designed for use with heterogeneous classes and are most successful when combined with in-depth, long-term professional development experiences for teachers. In addition to an integrated approach to content, they also incorporate the following components: rich tasks, alternative assessment, graphing calculator technology, higher-order thinking skills, and integration of reading, writing, and oral communication skills. Listings of these and other NSF projects are included in Chapter 6 and are also available on the SciMath$^{MN}$ webpage at www.informns.k12.mn.us/scimathmn.

A sample problem from one of the NSF sponsored curricula projects*, Interactive Mathematics Program* (IMP), is presented in Figure 3 (Fendel & Resek, 1999). In this problem, traditional content (solving systems of linear equations) is integrated with statistics, probability, curve fitting, and matrix algebra.

IMP units begin with a motivating problem that is too difficult for almost any of the students to solve immediately. Students examine this initial situation and then investigate similar and perhaps simpler situations. Throughout the unit students are required to actively pose questions, look for patterns, and make connections between the current problem and the mathematics they have learned in previous units. Through a variety of problems, their understanding expands and they begin to abstract concepts and refine techniques they can use in various ways to solve the original problem.

**Figure 3 - Sample of a problem from a high school integrated mathematics curriculum**

In the *Meadows or Malls?* unit found in Year 3 of the *Interactive Mathematics Program* (IMP), students are presented with the following problem:

You are on a planning team, consulting to the city manager. Your task is to come up with the best possible plan for the use of 550 acres of land recently obtained by the city. The acreage is comprised of a recently closed army base, a 300-acre farm, and abandoned mining land.

There are conflicting parties who are interested in the property. The business community is pushing for development schemes, while the environmental interests are advocating for more recreational space. The two factions have arrived at a partial compromise with which you have to work. They have agreed to the following two points:

- a maximum of 200 acres from the army base and the mining land will be used for recreation
- the amount of army land used for recreation plus the amount of farm land used for development will together total 100 acres.

Besides needing to meet the wishes of the opposing factions, you must deal with improvement costs ranging from $50 to $2000 per acre, depending on which parcel of land is involved and how it is going to be used. You have to satisfy everyone while minimizing the total costs for improvements.

(Problem from: *Meadows or Malls?* © 1999 by Interactive Mathematics Program. All rights reserved. Used with permission. Interactive Mathematics Program: Year 3, by Dan Fendel and Diane Resek with Lynne Alper and Sherry Fraser, published by Key Curriculum Press, P.O. Box 2304, Berkeley, CA 94702, 1-800-995-MATH.)

In the land use task, the complexity of the situation demands careful analysis and close attention to detail in order to arrive at an optimal allocation plan. To resolve the problem, students will need to solve systems of linear equations involving six variables, and use some complex geometrical reasoning involving the graphs of such equations.

Solving systems of linear equations for unknowns is an important skill in algebra. In IMP, this topic is developed and expanded over several units, including a Year 2 unit involving only two variables. Students don't just learn one method for solving equations. They develop many approaches in groups and share ideas with each other, building a varied repertoire of techniques.

An important part of the *Meadows or Malls?* unit is generalizing both the algebra and geometry beyond two variables. In doing so, students analyze the geometry of how two-dimensional planes intersect in three-dimensional space, see how to use matrices and graphing calculators to solve such systems, and develop and work with important abstract notions such as inverse and identity.

With several integrated mathematics projects now available, mathematics teachers finally have access to curricula that were written explicitly to model national standards. Because of the close parallel between the national mathematics standards and the *Minnesota Graduation Standards*, schools and district curriculum adoption teams should give serious consideration to integrated curriculum projects developed recently by NSF and various publishers (see listing in Chapter 6).

## K-12 Articulation

Teachers must be aware of not only the content in their course or grade level, but also how concepts are developed across the K-12 mathematics curriculum. Too often a spiral curriculum is reduced to a series of lessons that keep students reviewing previous concepts year after year at a superficial level. A well designed K-12 mathematics curriculum explicitly outlines the development of major concepts throughout a child's education so that new understandings can build on previous learning. By defining a curriculum that deliberately and systematically develops and assesses student understanding across grades K-12, it is more likely that the curriculum taught at each grade is developmentally appropriate and that the opportunity to learn concepts at more sophisticated levels occurs throughout a students' school experience.

*"Assessing a student's mathematical power requires many different indicators over time...it is important to ensure that measures are taken of a student's ability to reason in mathematical situations, to communicate perceptions and conclusions drawn from a mathematical context, and to connect the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation."*
NAGB, 1996

Chapter 3 of this *Framework* directly addresses K-12 articulation. For each content strand, the *Focus* section, along with the *Components* and bulleted key ideas, shows how a particular content area can be developed across several grades. For example, in *Shape, Space & Measurement,* children in the primary grades explore, compare, sort, and classify two- and three-dimensional objects in their physical world. In the intermediate grades, students will integrate measurement concepts, including area, perimeter, weight/mass, capacity, and angles with their exploration of shapes. Technological tools take advantage of and help students' develop their visualization skills. At the middle school level students will build on the generalizations from the intermediate grades to develop classifications of and definitions for shapes. Middle school students will begin to use shape as an analytical tool and use visualization, measurement, and proportional reasoning skills to develop understanding of the effects of scale change on measures of length, area, and volume. By high school, students are connecting algebraic and geometric concepts, formalizing concepts of shape and space in one and two dimensions through axiomatic systems, and analyzing geometric concepts in three dimensions. (See Shape, Space & Measurement *Focus* section on pp. 16-17, *Components* on pp. 20-25, and *Sample Scope and Sequence* in *Appendix A*.)

## Structure of Mathematics

Another type of connection that should develop for students of mathematics over their K-12 experience is an understanding of and appreciation for the uniqueness of mathematics as a discipline, irrespective of context. This exploration of the structure of mathematics happens very slowly for students and continues beyond K-12 mathematics into advanced-level study.

Jones and Bush (1996) identify two types of structures in mathematics: **conceptual structures** and **axiomatic structures**.

*"School tradition has it that arithmetic, measurement, algebra, and a smattering of geometry represent the fundamentals of mathematics. But there is much more to the root system of mathematics—deep ideas that nourish the growing branches of mathematics."*
Steen, 1990

"**Conceptual structures** comprise mathematical concepts, their definitions, their properties, and the relationships among them" (p. 718). One aspect of exploring conceptual structures is to investigate definitions, which are accepted by mathematicians as true. For example, a square can be defined as a "rectangle with congruent sides." It can also be defined as a "rhombus with one right angle." Are these equivalent? Is one definition better than another? Is one definition more useful than another? Students of mathematics can be encouraged to define concepts in multiple ways, critique and compare those definitions, and struggle with the difficulty of using language to express mathematical relationships.

Conceptual structures also serve to explicate the relationships among mathematical concepts. For example, when introduced to a new number system, students should be encouraged to investigate "what is gained, what is lost, and what is retained in the structural characteristics of each new system" (NCTM, 1989, p. 185). At the secondary level, students might be asked to organize the following terms to indicate relationships within the number system (Jones and Bush, p. 718):

| | | |
|---|---|---|
| • decimals | • whole numbers | • zero |
| • irrational numbers | • fractions | • real numbers |
| • integers | • percents | • counting numbers |
| • rational numbers | • pure imaginary numbers | • complex numbers |

Questions like these that focus students' attention on mathematical relationships are crucial if students are to develop an understanding of mathematics that goes beyond superficial parroting of information.

Concept structures can be represented in a number of ways, including Venn diagrams, tables, charts, webs or other types of mind maps. The challenge for students (and teachers as well) is the personal struggle to uncover and communicate the mathematical relationships involved.

*"Many who have never had the occasion to discover more about mathematics confuse it with arithmetic and consider it a dry science. In reality, however, it is a science which demands the greatest imagination."*
Sofia Kovalevskaya, 19th Century Russian mathematician

"**Axiomatic structures**…are composed of mathematical axioms, postulates, theorems, procedures, rules, formulas, and laws...The purpose of axiomatic structure is to formalize relationships between what is taken to be true about mathematical objects, that is, axioms and postulates; what can be proved about those objects through formal reasoning processes, that is, theorems; and what is used to solve problems, that is, procedures, formulas, and rules" (p. 716-717).

In mathematics, a statement is said to be true when a logical, step-by-step, coherent argument is provided; false if a counterexample to the statement is found; and neither true nor false otherwise. This criterion of mathematical truth is much more precise and formal than what most people generally take as a definition of truth, and it is this criterion that sets mathematics apart from the other sciences (Benson and Vessey, 1996, p. 5).

As students progress though the grades, they need to understand that while some mathematical statements are accepted by the mathematics community as true, others require evidence. Verification of mathematical truth should progress from informal observation to demonstration by example to conjecture to formal reasoning to proof. At the high school level, the study of mathematics with attention to axiomatic structure can provide students the opportunity to encounter reasoning in one of its purest forms and can prepare them for more abstract mathematics.

Concept and axiomatic structures are related and interdependent, and experience with both is necessary (Jones and Bush, p. 720). Each structure alone is limited, but together they can help students develop a deep understanding of mathematics and begin to appreciate and understand the "whys" of mathematics.

*" (To find mathematics in many cultures) one must search the literature of many disciplines — history, economics, ethnology, anthropology, archaeology, linguistics, art and oral tradition — and still be dissatisfied. African literature — novels, poems, essays — sometimes tells more about mathematics in the context of the lives of the people than do articles in learned journals."*
Zaslavsky, 1973

## Part 2: Connections between Mathematics and Other Disciplines

Traditional mathematics materials have stressed the history of mathematics from a European perspective which tends to compartmentalize learning into discrete subjects. Many other cultures, however, take a more holistic view of knowledge. Similar questions have been asked by various peoples at different times, for different reasons, across various disciplines, and their perspectives on the solutions have enriched the study of mathematics. Studying a concept across disciplines brings various perspectives into a unit of study and may be a way to teach students whose strength is in another discipline.

There are many mathematical concepts and skills that play a critical role in other disciplines. For example, science, mathematics, and social studies all require students to produce and interpret graphs. Yet, because the format, terminology, instruction, and grading are approached differently, graphing may be mistakenly perceived by students as a set of distinctly unique skills or processes within each discipline.

Concepts that are common or closely related across disciplines offer rich opportunities for interdisciplinary integration. Symmetry, networks, classification, balance, patterns, and use of variables are examples of powerful ideas that are represented in many disciplines. Establishing interdisciplinary connections, however, can be difficult since they require that teachers:

- be aware of and have some knowledge about the curriculum in other disciplines
- have an understanding of the fundamental skills and concepts in each discipline
- agree on what the skills and concepts mean

There are several ways to connect mathematics to other disciplines.

### Thematic Connections

The purpose of using a theme is to provide students with a motivating context within which to connect learning across many disciplines. Instruction and assessment should not be focused on students' understanding of the theme, but rather on the knowledge, skills, and understandings related to each of the disciplines involved.

*Bears* is a frequently used theme in the primary grades. Figure 4 shows an adaptation of a sample web from a class brainstorming session about bears, incorporating the questions and interests of primary students (Piazza, Scott, and Carver, 1994, p. 295).

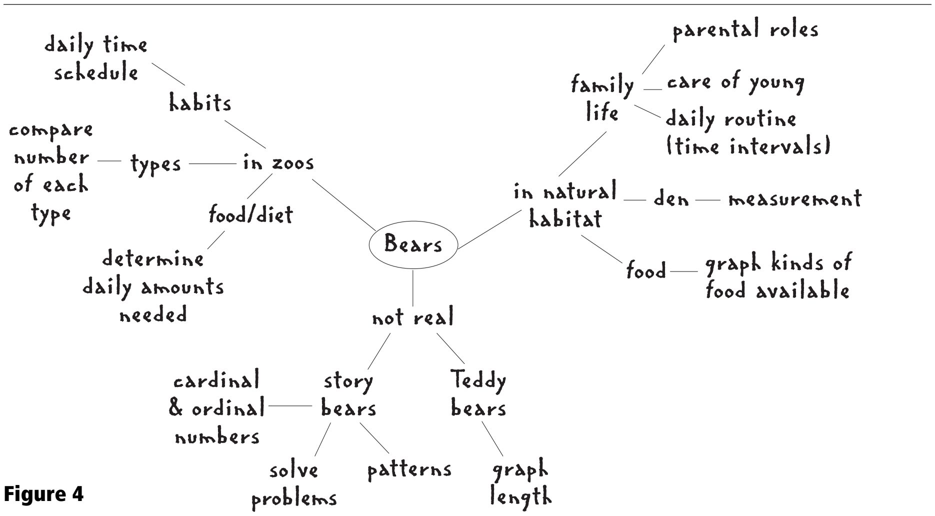



**Figure 4**

While opportunities arise for problem solving, measurement, data collection, and connections to real world situations, the mathematics in the *Bears* unit is not immediately evident. To more explicitly show the mathematical connections to this theme, Piazza, Scott, and Carver (p. 296) created mathematical learning activities from the ideas generated in the web. They connected the learning activities to the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) for Grades K-4. Table 2, excerpted from their work, shows some of these relationships.

**Table 2 – Relating the Study of Bears to the NCTM K-4 Standards**

| | |
|---|---|
| Problem Solving | Determine daily diet and amounts of food for zoo bears.<br>Research available food for bears in a given natural habitat. |
| Communication | Develop bear diet recipes.<br>Read factual information on bears and display information graphically. |
| Estimation | Estimate number of teddy-bear counters, gummy bears, etc.<br>Estimate weights and heights of bears in given photographs. |
| Number sense | Perform teddy-bear activities relating ordinal and cardinal numbers.<br>Depict daily routine of bears in a time schedule. |
| Measurement | Measure body parts of various teddy bears.<br>Determine measurements for a den. |
| Patterns and Relationships | Replicate, extend, and create patterns with teddy-bear counters, bear paws, and so on. |
| Statistics and Probability | Collect, organize, construct, read, interpret, and display data about kinds of bears in a variety of forms (object graph, picture graph, bar graph, etc.) |

*"We need to be careful that we don't get so excited about the 'context' that we lose sight of keeping 'content' standards in the curriculum."*
Hoye, 1997

Themes do not in and of themselves ensure good connections. In thematic instruction, the theme or situation is not what students are to learn, but simply something that holds ideas together for instruction. Declaring the theme of the month to be *apples* or *clowns* does not result in a cohesive curriculum that supports students' mathematical development. Themes are potential vehicles for building connections but many times the depth or significance of the mathematical content is sacrificed to serve the theme. It is important to evaluate thematic curriculum to make sure that time is not invested in efforts that are more entertaining than educational.

### Connections between Mathematics and Science

Mathematics and science have long been thought to be natural allies in thinking about the world. However, declaring a relationship between science and mathematics does not automatically ensure significant connections will be made and used in a constructive manner. One way to view the mathematics-science connection is to map concepts along a continuum. Lonning and DeFranco (1994) suggest that not every mathematics topic lends itself to a science application and not every science activity involves mathematics. This is shown in Figure 5 in their *Continuum Model for Mathematics and Science* which is an effective way to think about and make connections to both science and mathematics.

**Figure 5 - Continuum Model for Mathematics and Science**

| Independent Mathematics | Mathematics Focus | Balanced Mathematics and Science | Science Focus | Independent Science |
|---|---|---|---|---|
| These concepts are taught in a purely mathematical context. | What mathematics concepts are being taught? Are there science activities that support these concepts? | Activities which provide a natural integration of mathematics and science concepts. | What science concepts are being taught? Can appropriate mathematics concepts or skills be incorporated? | These concepts do not require or involve mathematics. |

For example, collecting and displaying data is common to both mathematics and science. Data from scientific inquiries could be analyzed in mathematics, allowing students to engage in real-life investigations. Explorations in the mathematics classroom help students develop skills in various forms of statistical displays. In both science and mathematics classrooms it is helpful to describe, analyze and interpret displays of data by asking questions such as the following: In which representations can the original pieces of data still be determined? What is the overall picture of the data? Would we expect to always get these results? How might the data change over time?

Symmetry involving reflections, rotations, and transformations can be examined within mathematics, while symmetries such as the helix or spiral can be part of scientific study. Students studying symmetry might identify where symmetry appears in nature and how symmetry affects the function of a given plant or organism.

*"Except for its relationship to science, mathematics is the forgotten cousin in interdisciplinary teaching and learning."*
Kleiman, 1991

Both science and mathematics use models to explain the world. Models may take many forms (physical, mathematical, conceptual) and are a means of helping students, scientists, and mathematicians explore and understand how things work. However, the *National Science Education Standards* (NRC, 1996) point out that even middle and high school students "view models as physical copies of reality and not as conceptual representations." If students are to gain facility with using models, they must have a rich variety of direct hands-on experiences, including, as the science benchmarks (AAAS, 1993) point out, "images and understandings that come from art, stories, sports, work, and living."

Measurement, changes in scale, and rates of change easily cross over both mathematics and science. Careful coordination between science and mathematics courses might give students multiple ways of looking at change as a natural phenomena.

The *Continuum Model for Mathematics and Science* suggests a way to take a balanced approach to connecting mathematics and science — two strongly related disciplines. In connecting mathematics to other subject areas, teachers may find the same continuum model helpful as a foundation for planning connections.

### Connections to Other Disciplines

An important mathematical concept can also serve as a connector to learning experiences in other disciplines. As students experience a concept across various disciplines, their understanding is reinforced or deepened. Interdisciplinary approaches give students several lenses for examining a concept and aid their ability to apply a concept in new situations.

*"With a real lens, you can look at the texture of wood grain, ants scavenging a beetle, the pupil of an eye. Likewise, a good integrative theme applies pervasively throughout a topic...reveals patterns fundamental to the subject matter...discloses fundamental similarities and contrasts within and across the disciplines...(and) fascinates teachers and students."*

Perkins, 1989

Interdisciplinary study is also an excellent way to embed cultural perspectives. Network theory (the study of vertices and the edges connecting them) is a field of discrete mathematics that is growing in importance with its applicability to solving real-life problems. (See *Discrete Mathematics* content strand in Chapter 3.) Networks are also elements in the artistic expression of various cultures and are designed and used in several fields of work. A unit on networks for intermediate or middle grade students might focus on the mathematics of networks but also connect that study to explorations in literature, visual arts, and real-life work experiences. In Figure 6, a curriculum planning wheel is used to design an interdisciplinary unit on networks. The unit is summarized in Figure 7. This planning wheel and summary show how a major concept in mathematics can be linked to different disciplines and real-life applications. Because the conceptual theme is the major focus, many important and carefully selected aspects of mathematics are linked to this conceptual theme and can be assessed through the projects in which the students are involved. (Part 4 of this chapter has more information on using a curriculum planning wheel.)

The curriculum planning wheel also aids in planning integrated assessment activities. For example, student understanding of the concept of traceability of a network can be evaluated by:

- taking familiar objects such as the letters of the alphabet and having students identify which are traceable networks and which are not
- identifying occupations that involve knowledge and use of networks
- determining an efficient network for a system of computers in a school setting

With an appropriate rubric for scoring, project work or performance tasks involved in the real-life applications of networks can be valuable assessment tools for the unit. The Performance Packages in the *Minnesota Graduation Standards* contain many related tasks that might be delivered across several disciplines. With careful and often cooperative planning, interdisciplinary units can provide a substantially connected approach to mathematics that enhances learning and students' ability to demonstrate what they know across several disciplines.

**Figure 6 - Planning for a Unit about Networks Using a Curriculum Wheel**

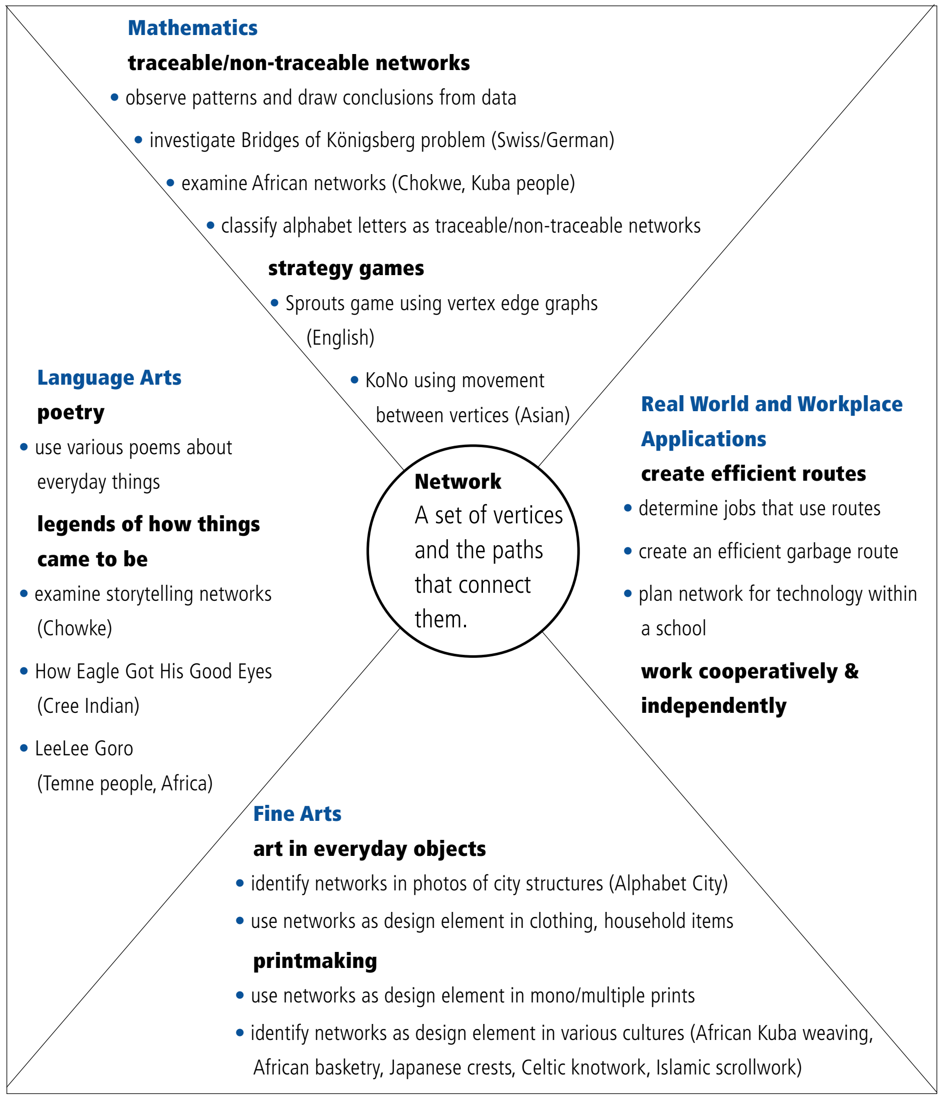
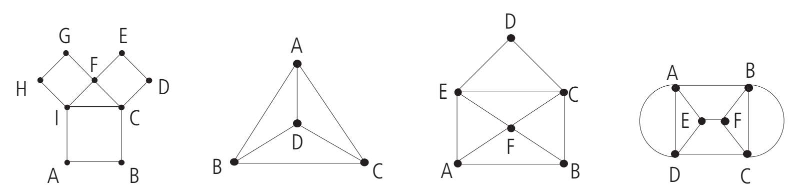


**Figure 7 - Networks: Connecting Mathematics to Other Disciplines and Cultural Perspectives**

In mathematics, a traceable network is a continuous line that connects several vertices. In traditional African storytelling, continuous lines were drawn in the sand around stones to connect the parts of the story. The designs drawn in the sand can be seen woven into fabrics and baskets and painted on pottery by African artists. A unit centered on the concept of networks might begin with the retelling of an African legend. The teacher draws a continuous line on the overhead or chart paper, connecting several points as the story is told. Students focus initially on the details of the story but are led to observe that the line drawn as the story is told is a continuous one. "You never picked up your marker" is a common explanation of continuity at this stage of study.

With the storytelling as a lead-in to the unit, students then explore various figures, like those shown below, to determine whether or not they are traceable in one continuous line that will go through all the vertices.




Students chart the success of their results and which vertices they enter and exit in order to trace the figure in a continuous line. Reflecting upon the data, students might draw conclusions similar to those of Leonard Euler, a Swiss mathematician of the 18th century, who established rules for networks in his proof of the impossibility of crossing the seven bridges of Königsberg on a continuous trip. (See *Reflections* section, *Discrete Mathematics* content strand, Chapter 3, for a more detailed discussion of the Königsberg Bridge problem.)

After understanding how vertices might be connected by a continuous line, students explore the idea of networks as routes for getting a job done. Using enlargements of sections of their city or town they attempt to determine an efficient route for garbage collection from each block within the designated area of their map. The optimum or most efficient path may have to be adjusted due to factors such as dead-end or one-way streets. In a real-life application, a true mathematical network may not exist.

Another use of networks is found in airline routes between hub cities. Observing the connections on a United States or world map for a particular airline introduces the idea of hubs. Several paths may go through a particular vertex (city) as long as the path between a given set of vertices (cities) is a continuous line. As an application, students might draw plans to network electronic equipment in their school, utilizing hubs and servers. These plans can serve as a performance assessment, indicating student understanding of a traceable network as a continuous path connecting all the vertices (workstations, hubs) under consideration.

As an extension, students might investigate the arts of various cultures that have networks as a main design element (e.g., African knotwork can be compared to Celtic knotwork). Many Japanese family crests use networks as a design element and the tiling patterns of several Arabic nations can show lattice-like patterns that are based on networks. Exploring a mathematical idea through the arts may be a gateway to mathematical understanding for those students whose learning style preferences are strongly demonstrated in the arts. This approach also expands the idea of a network for those students who are comfortable with the abstract idea in mathematics but gain a deeper understanding by applying the concept in practical situations.

(Summary of unit developed by N.A. Nutting, 1996, Andersen Multicultural Laboratory Demonstration Site, 2727-10th Avenue South, Minneapolis, MN 55407, phone: 612.627.3240.)

## Part 3: Application-based Connections

Interdisciplinary studies will often give rise to real-life applications of concepts. Knowing how a concept is applied in the real world increases not only the interest in, but the understanding of, a concept by students. Application-based connections are designed to help students relate their learning to issues and contexts outside of school. These connections provide students with the opportunity to learn and use their knowledge, skills, and understandings in authentic experiences.

*"All students bring to school mathematical knowledge gained from everyday situations they have experienced. However, such knowledge is often hidden and unused in school...Students need in-school mathematical experiences to build on and formalize the mathematical knowledge gained in out-of-school situations."*

Masingila, 1995

When mathematics is used in the world outside the classroom there are rarely algorithms or answers provided. Those who use mathematics in the workplace provide mathematical arguments designed to persuade others of their conclusions. Often one needs to come up with a function or formula that appears to model the data relatively well. This is most often a multi-step process in which attempts are made, then checked, then improved until a reasonably accurate model is developed. Then one attempts to apply this model to solve the problem in question. These activities, in general, form the heart of mathematics.

### Research Base for Applications

In *Defining Excellence for American Schools,* Daggett (1994a) compared curricula in America, Europe and Asia. These comparisons show that while American curricula help students reach the upper levels of *analysis, synthesis, and evaluation* in Bloom's Taxonomy quite well, there is very little teaching of how to apply knowledge (p. 38).

Daggett proposed the *Application Model* as a construct for developing curriculum. This model, shown in Table 3, is a continuum of five levels.

**Table 3 – Application Model (Daggett, 1994a)**

| Level | Description | Description applied to mathematics |
|---|---|---|
| 1 | Knowledge | Knowledge of mathematics for its own sake. |
| 2 | Apply in discipline | Ability to gather and use knowledge in mathematics |
| 3 | Apply across discipline | Ability to use mathematics in other subject areas |
| 4 | Apply to predictable situation | Ability to use mathematics in structured, known solution path situations |
| 5 | Apply to unpredictable situation | Ability to use mathematics in situations where the solution path is unknown and the conclusions are complex and unpredictable |

In curriculum design, Daggett believes that developers need to look first for important applications at levels 4 and 5 of the *Application Model* and then work backward to what knowledge is needed in a discipline. An example of using the *Application Model* within mathematics is shown in Table 4.

**Table 4 – The Mathematics of Vertex-Edge Graphs in Daggett's Application Model**

| Level | Description | Description applied to vertex-edge graphs |
|---|---|---|
| 1 | Knowledge | Students recognize that a vertex-edge graph is a mathematical structure consisting of vertices and edges in which some pairs of vertices are connected by edges. |
| 2 | Apply in discipline | Students generate tree diagrams to delineate all the outcomes of a probability experiment. |
| 3 | Apply across discipline | Students use vertex-edge graphs to diagram an electronic network and explore the benefits of various plans. |
| 4 | Apply to predictable situation | Students use vertex-edge graphs to solve the problem of scheduling club meeting days when some students belong to more than one club. Vertices represent clubs, edges represent any two clubs that share members. The graph is a vehicle to identify the existence of one or more solutions. |
| 5 | Apply to unpredictable situation | Students research the steps involved in starting up a business. They determine the steps that would be necessary and create a vertex-edge graph that shows the sequence of the steps, which steps must precede others, and steps where the order is not critical. Using critical path analysis techniques from graph theory, students determine the best path through the steps and determine minimum and maximum timelines for the startup project. |

*" America needs to make a critical decision about what it means to be educated. Are we concerned simply with students moving up on Bloom's Taxonomy to higher and higher levels of knowledge in a subject, or should we also be concerned with their ability to apply the knowledge they have learned?"*
Daggett, 1994a

In his research Daggett found that students in Asian countries excel at applying knowledge within a discipline, across disciplines, and in predictable situations, and do better than American or European students in applying knowledge in unpredictable situations. It is only in vocational education programs at the secondary level and in engineering, health, and business programs at the post-secondary level that American students achieve a high degree of applied knowledge. Daggett's analysis indicates that many American students are not gaining skills in gathering and using information and do not access and assimilate knowledge from a variety of sources to solve complex problems such as those found in the workplace (pp. 40-44). In an analysis of standardized tests, Daggett (1994b) found 0-2% of the problems used in the United States are application based. In Asian countries 85% of the problems on standardized tests center on applications.

Likewise, Steen in a recently prepared paper for SciMath^MN, *What Employers and Educators Test: The Mathematics that Really Counts* (1997), raises similar questions about the alignment of traditional assessment with the mathematical content, skills, and processes that students need in real life and in the workplace. Employer expectations do not reference parabolas, matrices, functions, or exponential growth but do expect sophisticated understanding of applied problems in the workplace. In addition, skills such as making decisions, allocating resources, communicating solutions with others, seeking multiple solutions to a problem, or producing a product are needed in the workplace. Interdisciplinary studies connected to real life situations can heighten the development and use of such skills. The mismatch that exists between how students are frequently tested in mathematics and the expectations of employers means that students and the public may be misguided about what mathematics is essential for students to take with them into the workplace.

### Real-life Connections within the Classroom

A wide range of applications can be found and used in the classroom to help students make connections to the use of mathematics outside of school.

- Running a school store or a similar project gives students a variety of mathematical experiences matched to the age level of the students.
- Analyzing a sales project from simple profit and loss calculations to spreadsheet-based business decisions, allows students to experience components of managing a business.
- Planning a school or community event involves many decisions about which plans may work out best and which are most cost efficient. Many items or services will have to be purchased and several factors may influence the decisions made.
- Researching careers and using data displays and other comparative mathematics to present their findings can also help students see the need for mathematics in their futures.
- Using geometric or other graphic software packages might allow students to experiment with changes in the design of an item such as a packing container or a new playground.

### Real-life Connections to the Workplace

*" What's missing from . . . exams are problems whose content reflects the special ways in which mathematics is used in contemporary jobs. There are no spreadsheets or quality control charts, no CAD/CAM systems or three dimensional graphics, no statistical significance or balanced design, no amortization tables or annuity formulas."*

Steen, 1997

In recent years, state and national attention has been drawn to school-to-work programs which stress the connections between school learning and workplace skills. There is a growing understanding that applied learning is essential for all students. It is a widely used instructional strategy in countries whose students demonstrate high achievement in mathematics. Curricula, such as those developed for school-to-work programs, teach mathematical concepts in the context of occupational experiences to make connections between the mathematics encountered in the classroom and the content and skills that are used in the workplace. In addition, invited speakers can share their stories and skills and describe how they use mathematics in their employment. Companies may produce activities that provide students with experiences that are representative of what takes place in the workplace.

Performance packages in the *Minnesota Graduation Standards* often include tasks that are derived from real-world applications. The Technical Application standard requires students to apply mathematical concepts to technological problems and/or the creation of new products. In designing a set of plans for and constructing a complex piece of furniture or cabinetry, students apply many concepts of measurement. The accuracy of their *bill of materials* list and the specifications indicated require a precise use of measurement tools and concepts, assisted by computational technologies.

In all mathematics classes, application-based connections can be a powerful tool for improving student understanding and providing students with an answer to the eternal question, "When am I ever going to use this?" However, it is essential that the central learnings from these applied experiences are rooted in appropriate and important mathematical content. The application, first and foremost, is a vehicle for teaching the mathematics. Teachers play a key role in focusing student attention on the mathematics in the experience and assessing students' understanding and ability to apply that mathematics to new situations.

## Part 4: Tools for Planning Curriculum Connections

Incorporating meaningful connections efficiently into the mathematics curriculum is a complex undertaking. The challenge is to seek the most appropriate contexts within which to learn mathematics while not forcing artificial connections.

This section contains three tools for instructional planning. **Concept mapping** is a tool that both teachers and students can use to identify and delineate relationships between the concepts in a unit of study. A **curriculum planning wheel** is a helpful brainstorming and organizing device that provides a balance among possible connections. A **team model for secondary interdisciplinary planning** can be used to create a focus for student work across several disciplines. This approach takes careful planning and cooperation to ensure that the major concepts of various disciplines are connected in a mutually beneficial way.

### Concept Mapping

Bartels (1995, p. 544) defines a concept map as "an instrument for explicitly describing concepts and the relationships among them." The mathematical ideas on a concept map are placed in shapes (typically ovals), and lines connecting the mathematical concepts identify the relationships. Words along the lines (usually verb phrases) succinctly describe the linkages. Concept maps can be either hierarchical (building from the most general concept at the top to increasing levels of specificity at the lower levels), or webbed (several concepts linked to each other to indicate relationships among them.)

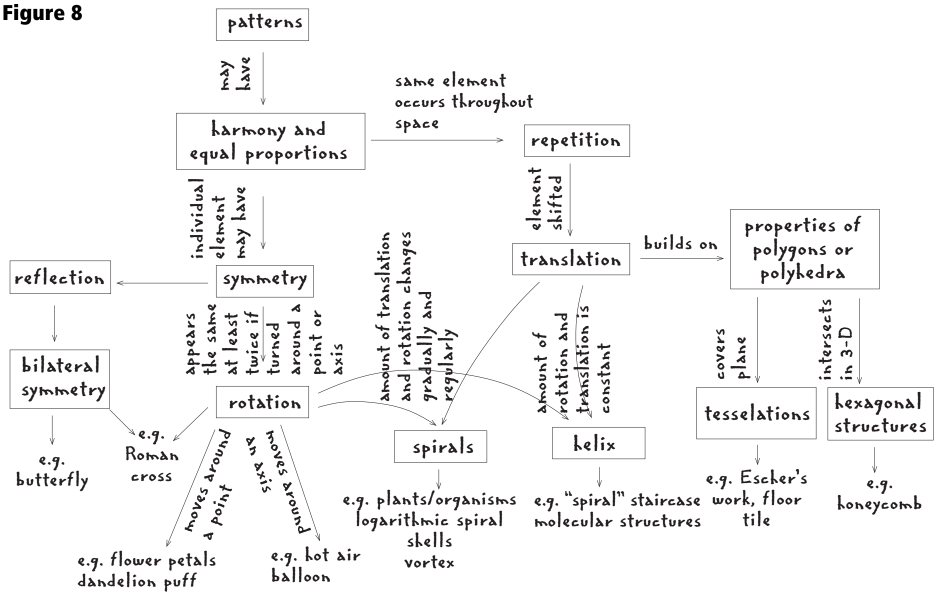
Teachers may find concept mapping useful in planning a unit of study. By identifying the key concepts and arranging them on a map indicating their relationships, an important guide for instructional planning is created. Concept maps are useful in generating relationships within a mathematical strand and ultimately help guide the teacher to direct students in making important connections with other disciplines and real life situations.

*"Typically, mathematical connections are implicit in instruction. That is, teachers use instruction that is connected but do not make the connections explicit for students."*
Bartels, 1995

In planning a unit on symmetry a teacher constructed the concept map shown in Figure 8. Using a supplementary resource material, *Symmetry: A Unifying Concept* (Hargittai & Hargittai, 1994), the teacher found many examples of symmetry in real life that stimulated examination of the complexity of symmetry and how frequently it is present in the world around us. By mapping the relationships and including examples of real-life applications, the teacher determined her view of the interrelationships and had a "road map" to guide the planning of the unit. Concept maps may vary from one person to the next. Another teacher's map might indicate different relationships to be emphasized in planning instruction for students. Even with similar maps, teachers might elect to sequence learning activities in different ways.

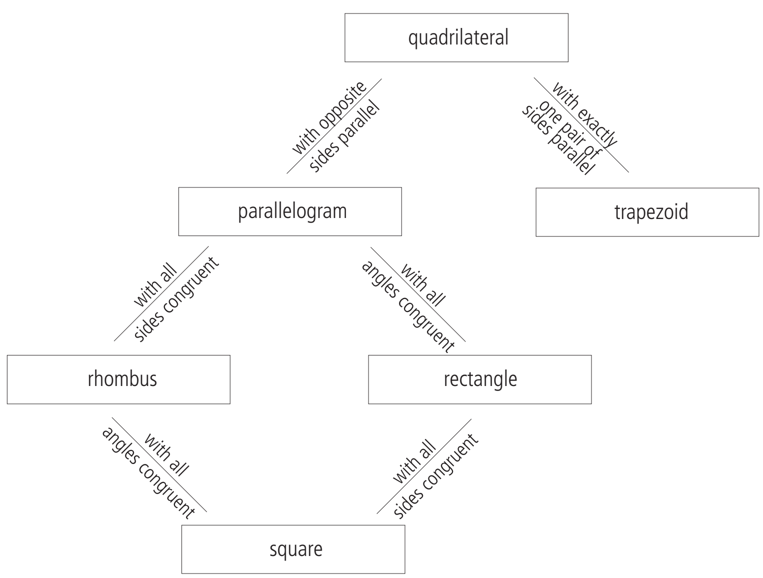
Concept mapping is also a way of helping students graph or show the relationships between and among concepts. Student-constructed concept maps give teachers valuable feedback on what students understand and become useful tools for adapting instruction. Farrell & Farmer (1988) and Bartels (1995) provide suggestions on how to help students become familiar with the logic involved in concept mapping. Novak & Gowin (1984) give step-by step procedures for various age levels to develop maps from students' experiences. Initially a teacher might present a completed concept map as a means of showing students the organization of a unit of study as well as the components of a concept map. Teachers might collectively map a lesson with their students to show the important relationships among the concepts of a lesson.

**Figure 8**




It may also be helpful to initially give students a concept map with empty shapes but with specific linkages indicated. Students would then insert concepts from a given list to indicate logical relationships. For example, students might be asked to make a map showing the connections among the following concepts: *parallelogram, quadrilateral, rectangle, rhombus, square, trapezoid*. A possible map is shown in Figure 9 (Geddes, 1992, p. 57).

**Figure 9**




*" (Concept maps are) an intriguing entry point into students' understandings."*
Hyerle, 1996

Students may also generate their own concept maps as a way to preview or review a lesson or unit. Students write each major concept from the lesson or unit on a separate index card or slip of paper. Students, individually or collaboratively, manipulate the cards or slips of paper on a large sheet of paper to explore the hierarchy or webbing that exists and the relationships they see among the concepts. They affix the individual cards or slips to the paper. They draw lines between related concepts and record key words along the lines to describe the relationships. Students, individually or collaboratively, might map concepts every few days during a unit, eventually creating a concept map for a complete unit of study. In fact, concept maps have been shown to be an effective study aid in preparing for a test (West, Farmer & Wolff, 1991, pp. 103-4).

*"Concept maps are powerful tools for observing the nuances of meaning a student holds for the concepts embedded in his/her map."*
Novak & Gowin, 1984

Concept maps drawn by students can be a useful form of assessment. Differences among maps can be analyzed to show conceptual misunderstandings or varying perspectives. These differences between and among maps often help students clarify conceptual understandings and see additional relationships. Student maps at the end of a unit of study are usually more elaborate and have more cross relationships. Evaluating whether the hierarchy or web and the corresponding relationships are correct, and which connections are not indicated or are not accurately described, can reveal the level of student understanding of the concepts and the direction for planning additional learning experiences.

Concept maps help all learners reflect on concepts and identify new relationships among concepts and can be particularly effective with students who have a visual learning style preference. The nature of the cross links and the quality of the hierarchy or web may indicate a level of creativity on the part of the student. The sophistication of a concept map will depend on the age and conceptual understanding of the students as well as the fluency that has developed from the amount and quality of previous experiences with concept mapping (Novak & Gowin, 1984, pp. 97-98). When collaboratively generated, maps require the strong use of communication skills in mathematics and may be a significant way of helping students verbalize the connections that exist in the study of mathematics. Whether concept maps are done individually or collaboratively, their power to engage students in mathematical reasoning is key.

### Curriculum Planning Wheel

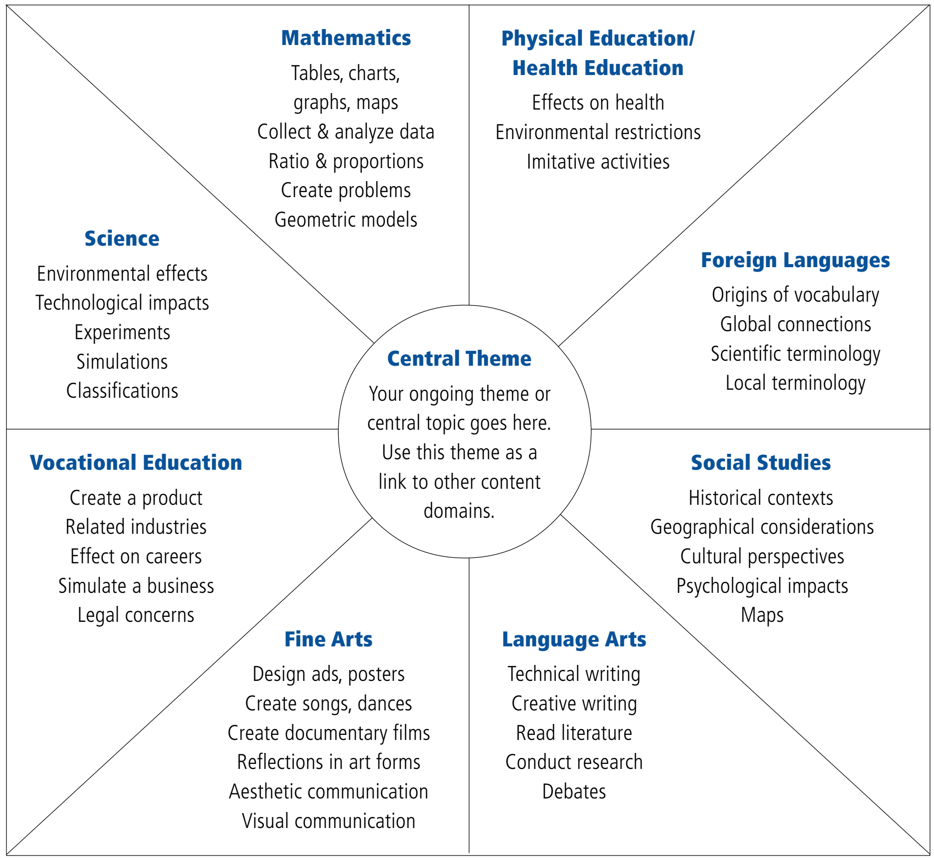
Interdisciplinary teaching increases flexibility in student thinking and allows students to examine ideas from different perspectives. By providing opportunities for students to learn a concept in other domains, McDonald and Czerniak (1994) believe the "relevance of ideas becomes clearer as the concepts are viewed from multiple perspectives and in greater depth" (p. 9). They have also noted an interaction between the disciplines that results in powerful ideas that come from the connections made. A planning wheel is a modified web and is one way to develop connected units of study.

McDonald and Czerniak developed the *Curricular Connections Planning Wheel* shown in Figure 10. Procedures for using their planning wheel include the following steps:

*"... the power may derive from the interplay of the disciplines in illuminating complex phenomena."*
Ackerman, quoted in McDonald & Czerniak, 1994

- identify a central theme, topic, concept, or issue that has important mathematics embedded in it
- identify the various content areas that could be connected with the central theme
- within each content area identify crucial concepts related to the central theme
- investigate the connections of the concepts across the disciplines (could be done using a concept map)
- develop specific activities and a sequence for instruction that best emphasizes the connections among the concepts of each discipline

**Figure 10 - Curriculum Connections Planning Wheel**




*" 'Where are we going to take our students?' In a sense, that is the ultimate design question curriculum makers should ask."*

Jacobs, 1989

### Team Model for Secondary Interdisciplinary Planning

As teams of teachers begin to look for ways to create connected units of study for students, they might employ the following four-phase method suggested by Jacobs (1991b). This model could be used in program decisions regarding how to deliver the *Minnesota Graduation Standards.*

#### Phase 1 - Conducting Action Research

Each member of a team of teachers plots his/her units of study month by month. Meeting together, members of the team analyze their plans looking for alignment of topics, how repetition can be eliminated or how a concept can be appropriately reinforced. They identify possible units for interdisciplinary study, especially those units which lend themselves to performance-based assessments. During the planning time team members should incorporate or seek out professional development opportunities that examine interdisciplinary curricular planning such as study groups, conferences, examining resources at a regional service center, university courses, site visits, publications of professional organizations, and reviews of literature from the larger education community. This phase may take place over several months.

#### Phase 2 - Developing a Proposal

Team members identify an area for interdisciplinary study, often choosing to upgrade an existing unit of study through collaboration among disciplines. If team members regard the unit of study as part of the core curriculum rather than an add-on, they will be more willing to spend the planning time needed. The planning should include evaluation, budget, timelines, and responsibilities of team members. Developing a proposal and planning for it may take several months of careful

*" Teachers need to be empowered with the skills and the time to examine what they're going to teach and how. Time is crucial...Common planning time is the necessary communication vehicle for teams."*
Jacobs, 1991a

thought. Time to pilot the unit (usually two to six weeks) should be set. Due to the critical importance of coordinated planning and careful monitoring during the pilot, a team may decide to schedule a pilot the following school year.

**Phase 3 - Implementing and Monitoring the Pilot**

Collegial collaboration is critical and often strengthened during the pilot phase. Data should be collected throughout the pilot regarding decisions made, time allotted for activities, adequacy of resources, political concerns, and impact on students. The data should drive discussions of revisions that might be necessary and the appropriateness and effectiveness of including the unit as part of the standard course of study.

**Phase 4 - Adopting the Program**

Decisions need to be made to elevate the pilot unit to a permanent part of the curriculum in order for it to be effective. Alignment to standards and the effect on student learning should be the main criteria in adopting a new unit of study. This may mean some other parts of the previous curriculum are eliminated or staff members change the time of the year to teach a particular concept so that students have a more integrated approach to a unifying topic of study.

An example of using this planning model to develop a unit on water use and conservation is shown in Figure 11.

**Figure 11 - Water Use and Conservation Unit**

*The Systemic Initiative for Montana Mathematics & Science (SIMMS) Project* materials integrate mathematics and science throughout the secondary school curriculum. Units were originally developed in collaboration with classroom teachers from Montana. A team of high school teachers who had adopted the SIMMS materials felt that their 9th grade students were especially successful with a Level 1 Unit called "What Will We Do When the Well Runs Dry?" (Montana Council of Teachers of Mathematics, 1993). Because of high student interest they approached staff members in other disciplines to make more powerful connections for students and to enhance the mathematics and science concepts in the unit of study.

In this mathematics unit, students examine personal water use, investigate concepts such as volume and rates of change, and learn to use linear equations resulting from the exploration of their data. Calculators and computers are used to help facilitate students' exploration of slopes, rates of change, and curves of best fit. Students use several formats to display the data. In science, changes produced by environmental and human factors are explored through investigations/experiments on water samples from various sources in the community. The water cycle is studied so students understand its components and what might occur when it is affected by various environmental and human influences.

In talking and planning with colleagues, the unit has now evolved to coordinate with the study of water in geography and its impact on a community. Comparisons of water use are made among various municipalities or counties and the effect of climate on water is explored. In English class, the unit on essay writing was moved to coincide with the teaching of the water unit so students could generate persuasive arguments for water conservation. A simulation of a community forum was established as the performance assessment for the unit of study. A current problem with water is identified and student groups debate several components of the issue using data displays, written information, oral arguments, models, and persuasive techniques to present their groups' perspective and solution of the problem.

## Summary

Brain research, learning theory, personal experience, and employers' expectations in the workplace all support the notion that making connections is essential to enhancing the mathematical learning of students. The challenges in making connections within mathematics, across discipline areas, and to real-life experiences should not result in either shallow connections or in paralysis. The ultimate question is: Does this change improve the mathematical understanding of students?

*" As students and teachers continue to 'think connections,' the connectedness of the mathematics will grow and become dominant. When that occurs, all will wonder why anyone had ever thought of mathematics in any other way."*
Coxford, 1995

The strategies in this chapter for helping students make mathematical connections are not a panacea for the many inherent difficulties in teaching mathematics. Teaching mathematics is not easy. Connecting mathematics will not automatically remove these difficulties. Making connections will not decrease the effort needed by learners to understand basic concepts of mathematics within a variety of settings. Learning mathematics takes effort and requires thinking, practice, and discussion.

Viewing mathematics as the science and language of connecting patterns, however, opens a wealth of opportunities for the learner and teacher. While there are difficulties and concerns in establishing these linkages, the power of understanding the connectedness of mathematics is much more important than the potential liabilities.

Making connections within mathematics, to other disciplines, and to real-life experiences allows more students to successfully enter into the exploration of problems to better understand what it means to do mathematics. Mathematics instruction that is rich in connections can lay important groundwork for this understanding of mathematics as a dynamic discipline and provides an essential foundation for students' future world of work.

## References

American Association for Advancement of Science (AAAS). (1993). *Benchmarks for scientific literacy.* Washington, D.C.: Author.

Association for Supervision and Curriculum Development (ASCD). (1991). "Focus issue: Integrating the curriculum." *Educational Leadership*, 49(2).

Bartels, B.H. (1995). "Promoting mathematics connections with concept mapping." *Mathematics Teaching in the Middle School*, 1(7), 542-549.

Beane, J. (1991). "The middle school: The natural home of integrated curriculum." *Educational Leadership,* 49(2), 9-13.

Benson, S. & Vessey, T. (1996). *Structure of mathematics.* Paper prepared for SciMath[MN], St. Paul, MN.

Caine, R.N. & Caine, G. (1990). "Understanding a brain-based approach to learning and teaching." *Educational Leadership,* 48(2), 66-70.

Clements, D. & McMillen, S. (1996). "Rethinking 'concrete manipulatives.'" *Teaching Children Mathematics,* 2(5), 270-279.

Coxford, A.F. (1995). "The case for connections." In P.A. House (Ed.), *Connecting mathematics across the curriculum*: *1995 Yearbook.* Reston, VA: NCTM.

Daggett, W.R. (1994a). *Defining excellence for American schools.* Schenectady, NY: International Center for Leadership in Education, Inc.

Daggett, W.R. (1994b). *Standards: An international perspective.* (Videotape). Schenectady, NY: International Center for Leadership in Education, Inc.

Eisner, E.W. (1997). "Cognition and representation: A new way to pursue the American dream?" *Phi Delta Kappan,* 78(5), 349-353.

Farrell, M.A. & Farmer, W.A. (1988). *Secondary mathematics instruction: An integrated approach.* Providence, RI: Janson Publications.

Fendel, D. & Resek, D. with Alper, L. & Fraser, S. (1999). "Meadows or malls?" *Interactive Mathematics Program: Year 3.* Berkeley, CA: Key Curriculum Press.

Fogarty, R. (1991). "Ten ways to integrate curriculum." *Educational Leadership,* 49(2), 61-65.

Geddes, D. (Ed.). (1992). *Geometry across the middle grades: Curriculum and evaluation standards for school mathematics addenda series, grades 5-8.* Reston, VA: NCTM.

Hargittai, I. & Hargittai, M. (1994). *Symmetry: A unifying concept.* Bolinas, CA: Shelter Publications.

House, P.A. (Ed.). (1995). *Connecting mathematics across the curriculum*: *1995 yearbook.* Reston, VA: NCTM.

Hoye, J.D. (1997, Fall). "Making school-to-work work." *Curriculum Update: Newsletter of Association for Supervision and Curriculum Development (ASCD).*

Hyerle, D. (1996). *Visual tools for constructing knowledge.* Alexandria, VA: Association for Supervision and Curriculum Development.

Jacobs, H.H. (Ed.). (1989). *Interdisciplinary curriculum: Design and implementation.* Alexandria, VA: Association for Supervision and Curriculum Development.

Jacobs, H.H. (1991a). "On interdisciplinary curriculum: A conversation with Heidi Hayes Jacobs." *Educational Leadership,* 49(2), 24-26.

Jacobs, H.H. (1991b). "Planning for curriculum integration." *Educational Leadership,* 49(2), 27-28.

Jones, D. & Bush, W.S. (1996). "Mathematical structures: Answering the "why" question." *Mathematics Teacher,* 89(9), 716-722.

Kleiman, G.M. (1991). "Mathematics across the curriculum." *Educational Leadership,* 49(2).

Kleiman, G.M. (1995). "Seeing and thinking mathematically in the middle school." In P.A. House (Ed.), Connecting *mathematics across the curriculum: 1995 yearbook.* Reston, VA: NCTM.

Lesh, R. (1979). "Mathematical learning disabilities: Consideration for identification, diagnosis, and remediation." In R. Lesh, D. Mierkiewicz & M.G. Kantowski (Eds.), *Applied mathematical problem solving.* Columbus, OH: ERIC/SMEAR.

Lonning, R.A. & DeFranco, T.C. (1994). "Development and implementation of an integrated mathematics/science preservice elementary methods course." *School Science and Mathematics,* 94(1), 18-25.

Masingila, J.O. (1995). "Carpet laying: An illustration of everyday mathematics." In P.A. House (Ed.), *Connecting mathematics across the curriculum: 1995 yearbook.* Reston, VA: NCTM.

McDonald, J. & Czerniak, C. (1994). "Developing interdisciplinary units: Strategies and examples." *School Science and Mathematics*, 94(1), 5-10.

National Assessment Governing Board (NAGB). (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress.* Washington, D.C.: U.S. Government Printing Office.

Montana Council of Teachers of Mathematics. (1993). *Systemic initiative for Montana mathematics & science (SIMMS) project: Level 1, volume 2.* Missoula, MT: Author.

National Center for Education Statistics (NCES). (1996). *Pursuing excellence: A study of U.S. eighth-grade mathematics and science teaching, learning, curriculum, and achievement in international context.* Washington, D.C.: U.S. Government Printing Office.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Research Council (NRC). (1996). *National science education standards*. Washington, D.C.: National Academy Press.

Novak, J.D. & Gowin, D.B. (1984). *Learning how to learn.* New York, NY: Cambridge University Press.

Nutting, N.A. (1996). *A series of lessons with networks: Making cultural and mathematical connections.* Minneapolis, MN: Andersen Multicultural Laboratory Demonstration Site.

Palmer, J.M. (1991). "Planning wheels turn curriculum around." *Educational Leadership,* 49(2), 57-60.

Perkins, D.N. (1984). "Selecting fertile themes for integrated learning." In H.H. Jacobs (Ed.), Interdisciplinary *curriculum: Design and implementation.* Alexandria, VA: Association for Supervision and Curriculum Development.

Piazza, J.A., Scott, M.M., & Carver, E.C. (1994). "Thematic webbing and the curriculum standards in the primary grades." *Arithmetic Teacher,* 41(6), 294-298.

Post, T.R. (1992). "Some notes on the nature of mathematical learning." In T.R. Post (Ed.), *Teaching mathematics in grades K-8: Research based methods.* Needham Heights, MA: Allyn and Bacon.

Schmidt, W.H., McKnight, C.C., & Raizen, S.A. (1997). *A splintered vision: An investigation of U.S. science and mathematics education.* Boston, MA: Kluwer Academic Publishers.

Steen, L.A. (1997). *What employers and educators test: The mathematics that really counts*. Paper prepared for SciMath^MN, St. Paul, MN.

Steen, L.A. (Ed.). (1990). "Preface." *On the shoulders of giants: New approaches to numeracy.* Washington, D.C.: National Academy Press.

West, C.K., Farmer, J.A., & Wolff, P.M. (1991). *Instructional design: Implications from cognitive science.* Needham Heights, MA: Allyn & Bacon.

Zaslavsky, C. (1973). *Africa counts: Number and pattern in African culture.* Brooklyn, NY: Lawrence Hill Books.