



The Oxford English Dictionary (1989) traces the development of the term “standard” to medieval times, when soldiers quickly lost sight of their leaders.

When going into battle, leaders began to carry with them “standards” - tall stakes to which a pennant was attached.

During times when neither side could tell who belonged to whom, leaders planted these standards, which by virtue of their height, stood out in the landscape, provided a direction for the soldiers, and showed what ground had been captured.

# Overview



### Intent and Purpose of the Minnesota K-12 Mathematics Framework

The *Curriculum and Evaluation Standards for School Mathematics*, developed by the National Council of Teachers of Mathematics (NCTM), and the *Minnesota Graduation Standards* are valuable documents for setting high standards for all children. The *Minnesota K-12 Mathematics Framework* is intended to build a bridge between these two sets of standards. This *Framework* provides a structure upon which to build the focused and coherent instruction necessary for student learning to occur.

### Background—How the Document was Developed

This document identifies six pervasive content strands of mathematics across the K-12 curriculum:

- Number Sense
- Shape, Space & Measurement
- Data Investigation
- Randomness and Uncertainty
- Patterns, Relationships & Functions
- Discrete Mathematics

It is essential that *all* children have experiences in each of these content strands every year as they move through school. It is also essential that the experiences be focused, sequential, and progress to deeper mathematical understanding at each level of instruction. The alignment of the six content strands with the NCTM *Curriculum and Evaluation Standards for School Mathematics* and the *Minnesota Graduation Standards* follows this overview.

To make this document useful and provide a focus for instruction, five grade level clusters were established:

- Primary (K-2)
- Intermediate (3-5)
- Middle School (6-8)
- High School (9-12; Graduation Standard Level)
- High School (9-12; Beyond Graduation Standard)

Within each content strand, key learnings are identified at each grade cluster to provide focused goals as well as learning that develops gradually from concrete to more abstract levels across the K-12 spectrum. The fifth cluster, High School (9-12; Beyond Graduation Standard) was included to show the level of advanced work that students in grades 9-12 might do that is currently not included in the *Minnesota Graduation Standards*.

With content and grade cluster parameters in place, a series of writing conferences was held from 1995-1996, each focusing on one of the six content strands. Educators across grades K-12 and education faculty from higher education institutions gathered to reflect on what Minnesota students should know and be able to do in that content strand. Participants were given readings from professional journals to study in advance. The work session began with a keynote address by an expert in the field. This expert remained available as a resource throughout the conference. This format allowed the expertise of Minnesota educators to help shape and inform this document in each of the content strands. To build consensus on what Minnesota students should learn and be able to do, participants reflected on two key questions:

*"We need to issue compasses and think more about instruction and staying on course."*

Ed Hessler, Executive Director, Minnesota Environmental Sciences Foundation, Inc.

What do you expect students entering your grade level cluster to know and be able to do?

What are you willing to be held accountable for teaching and students learning?

After brainstorming in small, grade level cluster groups, participants shared their ideas across K-12 and came to consensus on the “big ideas” that permeate the given content strand. The “big ideas” were identified as essential for organizing instruction and became the learning goals and components in the K-12 grid of each content strand. (See pages 6 and 7 in this section). Those ideas were then further developed to provide the bulleted ideas that indicate what students should know and be able to do.

### Organization — What Sections Comprise Each Content Strand?

Like their students, teachers learn in various ways. Therefore, each content strand of Chapter 3 has several sections to help teachers explore the content expectations and see how instruction might look in actual teaching situations.

Each content strand begins with the **Reflections** section. This is a summary of why the mathematics is important, what major concepts frame the content, and how the content should be taught. This information is drawn from the current research in the content strand as well as notes from the writing conferences and reviews by content experts.

Two kinds of **Vignettes** follow. The **Classroom Vignette(s)** shows an aspect of this content in a sample classroom lesson. The classroom vignette attempts to paint the vision of national and state standards at the classroom level where the payoff occurs. Elements of an effective classroom are shown, including organization of the classroom, use of contextual problems, purposeful teacher/student discourse, use of an inquiry/problem-centered approach, and attention to students’ reactions to the mathematics of the lesson. The **Reality Check** shows how a particular content strand of mathematics is applied in the workplace. These work vignettes are not intended as classroom lessons, but serve as reminders to classroom teachers that mathematics is a necessary tool in the workplace.

The **Focus** section presents a view of how the important concepts of each content strand develop over the K-12 curriculum. It is intended to suggest a focus for instruction within a specified grade level cluster as well as the development of the components of a content strand over time.

The **K-12 Components** are the blueprints with which local districts can build their curriculum and a guide to anchor teachers’ instructional planning. Each content strand has three to five components with key ideas, indicated by bullets, that are presented in the columns. The components are the main ideas in each content strand that are essential for planning. The components can also guide staff development. The subpoints give delineation to the component, showing more specific concepts in which to engage students and to assist instruction toward the learning goal. Each grade cluster is correlated to the *Minnesota Graduation Standards*. This correlation includes both mathematical standards and, where space allows, standards from other content areas of the *Minnesota Profile of Learning* that are supported by the mathematical content and processes in the columns.

**Sample Tasks** were recommended for each grade level cluster by the writing conference participants. These tasks were chosen from materials that were found to be helpful in teaching the content in a problem solving context and that would model an important aspect of the learning in these content strands. They are not included as an endorsement of any published materials, but rather as examples of what an individual lesson in a standards-based curriculum might look like.

The notes in each lesson provide insights about the delivery of individual lessons and how to pull important concepts out of students' explorations. Additional sources that participants identified are indicated for each grade level cluster.

The development of each section is supported by research and the reflective thinking of leaders in the content strand. Sources are cited in the **References** section.

In the Appendix of this document there is a **Sample K-12 Scope and Sequence** by specific grade level for one content area, Shape, Space & Measurement, to serve as an example of how this framework might be used in planning instruction at a specific grade level.

### Implementation — Where We Go From Here

This framework is only as useful as the professional conversations that occur among teachers, administrators, and community members about what is important for students to know and be able to do in mathematics and how teaching has to change for this learning to occur. It is hoped this framework will be a tool in initiating and supporting those conversations at state, district, school, and classroom levels.

Some important discussion issues are:

- How does this framework take learning in new directions?
- How has this framework been influenced by national standards for mathematics, *Minnesota's Graduation Standards*, and best practice in the teaching of mathematics?
- What are we already doing to support these recommendations?
- What changes need to occur in the local district's curriculum and the daily and year long planning of individual teachers or teams of teachers?
- What changes do we need to make in our professional development plans to ensure *all* children learn mathematics?
- What communication needs to happen so *all* stakeholders in education will remain on the course of high standards for *all* students?

The *Minnesota K-12 Mathematics Curriculum Framework* is intended to help districts focus on what needs to happen to raise the mathematical achievement of *all* Minnesota students. Engaging conversations, quality and sustained professional development, and careful planning within the state, district, community, and school will be necessary to make this happen. This framework gives structure for providing children meaningful experiences with important mathematics in a way that makes sense to them and from which they can make sense of mathematics.

**Minnesota K-12 Mathematics Curriculum Framework  
Summary of Learning Goals and Components**

October 1997

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**Number Sense**

STUDENTS will experience numbers in contextual situations and use them flexibly to solve problems, gradually moving to a more abstract understanding of number concepts, relationships and procedures.

With this learning goal in mind, Minnesota students will have the opportunity to:

- develop understandings about numbers, their magnitude and their relationships
- develop an understanding of numeration systems and their properties
- translate among equivalent forms of numbers to facilitate problem solving
- develop operation sense and use it to estimate results of operations
- understand the operations and apply them to solve problems

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**Shape, Space and Measurement**

STUDENTS will develop visualization and reasoning skills by investigating space, measurement concepts, and axiomatic systems, using appropriate tools and technologies.

With this learning goal in mind, Minnesota students will have the opportunity to:

- recognize, describe, and model shapes and relationships, and represent and reason about them in increasingly abstract ways
- use transformations and concepts of location, relative position, and symmetry to visualize, represent, and verify geometric relationships
- use measurement to make the connection between number and space in order to describe, analyze, model, and represent real world and abstract situations

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**Data Investigation**

STUDENTS will ask questions and collect, organize, interpret, and transform data related to those questions to communicate information and make predictions and decisions.

With this outcome in mind, Minnesota students will have the opportunity to:

- formulate a question(s), determine necessary data, and choose an appropriate method of data collection in order to make sense of a problem or situation
- collect, organize, and represent data in a variety of ways
- read, describe, and interpret displays of data
- formulate, justify, and communicate conclusions, arguments, predictions, decisions, and further investigations based on data

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**Randomness and Uncertainty**

STUDENTS will apply concepts of randomness and uncertainty to make critical judgments, predictions, and decisions about probabilistic situations.

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With this outcome in mind, Minnesota students will have the opportunity to:

- develop an intuitive sense of probability, use probability as a measure of uncertainty, and develop and use the language of probability to communicate
- model situations involving uncertainty by designing and carrying out experiments or simulations to estimate probabilities, solve problems, and make predictions
- create and use different types of distributions as a basis for making inferences or predictions about outcomes or populations
- explore current problems and events involving uncertainty and recommend or justify a course of action when appropriate

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**Patterns, Relationships & Functions**

STUDENTS will represent, compare, and analyze mathematical patterns, relationships and functions to model and solve problems.

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With this learning goal in mind, Minnesota students will have the opportunity to:

- recognize, describe and generalize patterns and build mathematical models to make predictions
- analyze the interaction between quantities and/or variables to model patterns of change
- use algebraic concepts and processes to represent and solve problems that involve variable quantities

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**Discrete Mathematics**

STUDENTS use discrete structures to model and understand mathematical relationships and solve problems.

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With this outcome in mind, Minnesota students will have the opportunity to:

- investigate and apply systematic counting techniques, set relationships, and principles of logic to represent, analyze, and solve problems
- use charts, vertex-edge graphs, and matrices to model and solve problems.
- explore, develop, and analyze algorithmic thinking to accomplish a task or solve a problem
- analyze, extend, and model iterative and recursive patterns

**Mathematics Alignment Across National and State Documents**

<b>NCTM Curriculum and Evaluation Standards</b>	<b>MN K-12 Mathematics Framework</b>	<b>MN Graduation Standards</b>
<b>K-4 Standards</b>	<b>Primary/Intermediate Grade Clusters</b>	<b>Primary Standards</b>
1. Problem Solving	Integrated throughout the framework	Integrated throughout the standards
2. Communication		
3. Reasoning		
4. Connections		
5. Estimation	Number Sense	Number Sense
6. Number Sense	Discrete Mathematics	
7. Concepts of Whole Numbers		
8. Whole Number Computation		
12. Fractions and Decimals		
9. Geometry and Spatial Sense	Shape, Space & Measurement	Shape, Space & Measurement
10. Measurement		
11. Statistics and Probability	Data Investigation Randomness and Uncertainty	Data Categorization, Classification, and Recording (Inquiry)
13. Patterns and Relationships	Patterns, Relationships & Functions	Shape, Space & Measurement
	Discrete Mathematics	Number Sense
<b>5-8 Standards</b>	<b>Intermediate and Middle School Grade Clusters</b>	<b>Intermediate and Middle Level Standards</b>
1. Problem Solving	Integrated throughout the framework	Integrated throughout the standards
2. Communication		
3. Reasoning		
4. Connections		
5. Number and Number Relationships	Number Sense	Number Sense (Intermediate)
6. Number Systems, Number Theory	Discrete Mathematics	Number Sense (Middle)
7. Computation and Estimation		
12. Geometry	Shape, Space & Measurement	Shape, Space & Measurement (Intermediate)
13. Measurement	Discrete Mathematics	Shape, Space & Measurement (Middle)
10. Statistics	Data Investigation	Chance and Data Handling (Intermediate)
		Chance and Data Handling (Middle)
11. Probability	Randomness and Uncertainty	Chance and Data Handling (Intermediate)
	Discrete Mathematics	Chance and Data Handling (Middle)
8. Patterns and Functions	Patterns, Relationships & Functions	Shape, Space & Measurement (Intermediate)
9. Algebra		Number Sense (Intermediate)
		Patterns and Functions (Middle)

**NCTM Curriculum and Evaluation Standards**

**9-12 Standards**

- 1. Problem Solving
- 2. Communication
- 3. Reasoning
- 4. Connections

- 7. Geometry from a Synthetic Perspective
- 8. Geometry from an Algebraic Perspective

10. Statistics

11. Probability

- 5. Algebra
- 6. Functions
- 9. Trigonometry

12. Discrete Mathematics

13. Conceptual Underpinnings of Calculus

14. Mathematics Structures

**MN K-12 Framework**

**High School (at Graduation Standard Level)**

Integrated throughout the framework

Number Sense

Shape, Space & Measurement

Data Investigation

Randomness and Uncertainty

Patterns, Relationships & Functions

Discrete Mathematics

Patterns, Relationships & Functions

Mathematics Structures (within Chapter 4: Connections)

**MN Graduation Standards**

**Graduation Level**

Integrated throughout the standards

Integrated throughout the standards

Shape, Space & Measurement  
Technical Applications

Chance and Data Analysis

Chance and Data Analysis

Algebraic Patterns  
Technical Applications

Discrete Mathematics

Algebraic Patterns  
Mathematical Research (Inquiry)  
Mathematical Research (Inquiry)

