

The Case for Change

“ No longer can we afford to sit idly by while our children
move through school without receiving mathematical
preparation appropriate for the twenty-first century...
The challenges are clear. The choices are before us.
It is time to act.”

NRC, 1989

We Believe...

The *Minnesota K-12 Mathematics Framework* represents the work of more than four hundred Minnesota mathematics educators K-16. Six beliefs about mathematics education served as guiding principles in this work. It is our hope that these beliefs will provide the direction for mathematics education in Minnesota into the next century.

All students can learn challenging mathematics and must be provided the opportunity to do so.

We believe all students, regardless of age, gender, race/ethnicity, socio-economic level, disability, primary language, or interest can demonstrate progress toward high standards when challenged by a rigorous curriculum delivered by teachers with strong content knowledge and flexible teaching strategies within a safe and supportive learning environment.

Mathematics standards can provide the alignment of curriculum, instruction, and assessment necessary to provide focus and coherence in Minnesota’s K-12 classrooms.

Standards are public statements about what all students should know and be able to do. Clear and useful statements about sound mathematics provide uniform criteria by which to judge student achievement. When curriculum, instruction, and assessment are aligned to standards, all children benefit. Standards make equity and excellence possible.

Learning mathematics is an active, collaborative process.

The process of collaborative inquiry is central to learning and understanding mathematics. Students must learn the content and processes of mathematics in a “hands-on, minds-on” approach by investigating problems, making connections to previous knowledge, predicting or conjecturing about possible solutions, communicating their reasoning, justifying solution strategies, generalizing their results, and applying their learning to new situations. Problem solving, both in school and in the workplace, is often best done by groups of people with diverse knowledge and experiences. Working collaboratively enhances learning, supports students’ mathematical conversation, and prepares students for life-work.

The integration of technology into mathematics curriculum, instruction, and assessment is essential.

Calculators and computers are necessary and useful tools for making mathematics accessible to all students, for learning important mathematics, and for preparing students for careers in an information society. Students should have regular and appropriate access to a variety of technological tools throughout their K-12 mathematics program.

Teachers are key to achieving excellence in mathematics education.

What happens in the classroom between teacher and student is the single most important element in the education system. We must treat teachers as professionals. They must be supported throughout their careers with meaningful and effective professional development opportunities, time, and appropriate resources in order to improve their practice and maximize student learning.

Public support for standards-based mathematics and increased student achievement is critical for improving and sustaining quality mathematics education.

Educators cannot implement standards alone. Everyone has a stake in improving students’ learning of mathematics, especially parents, community members, and business leaders. Every community member must make a commitment to student achievement by taking responsibility for implementing policies and practices that promote equity and excellence in mathematics education.

Chapter 1

Sometimes it is easier to see the “big picture” by looking at the landscape of reform from a different angle...

Make yourself comfortable in a large overstuffed chair. Close your eyes and breathe deeply for a few minutes. As you relax, concentrate your thoughts on a child who is very special to you. It might be your own child—a toddler, a teen, or even an adult child. It might be a grandchild or godchild, niece or nephew, or special neighborhood friend. The child you see in your mind’s eye might even be one of those students from the past who will always be special to you.

Develop the details in that image. Picture what that treasured child is wearing and doing. Listen for his/her spoken conversation and verbal mannerisms. Watch for those facial expressions that speak louder to you than words. Hold that child in your mind as you...

- [see her in a science class](#). How would you feel if that child whose existence you marvel at was never awed by the lives of other organisms? was never encouraged to wonder about how or why something worked? never used science equipment or did a science experiment? never learned about recent scientific discoveries?
- [see him in a French language class](#). How would you feel if that child who brings you so much joy was expected to only memorize and regurgitate French vocabulary and grammar in his course-work? never learned to speak the language or have a conversation? never had exposure to French culture?
- [see her in an art class](#). How would you feel if that child who colors your days did all her work in pencil, never getting to work with crayons, markers, pastels, acrylics, or watercolors? worked only in two dimensions and never had exposure to ceramics or sculpture or mobiles? never was allowed to create an original piece of art?
- [see him in an auto mechanics class](#). How would you feel if that child who is always on the go never got under a car or inside the hood for a closer look? never learned how the parts of the engine worked together to make the car run smoothly? never learned to diagnose a problem and repair it?

We would not tolerate these unreasonable approaches to learning science, art, foreign language, or especially auto mechanics, but these are exactly the practices we have allowed to dictate mathematics coursework:

- rote memorization of facts and algorithms with little concern for understanding why they work
- extensive classroom time devoted to “seatwork”
- rewards for classroom silence rather than mathematical conversation
- classrooms bereft of tools for doing mathematics
- absence of active, hands-on inquiry experiences
- few connections between mathematics and the natural world or between mathematics and other disciplines
- absence of any mathematical history, including its cultural influences

One reason these practices don’t provoke us is because, as students ourselves, most of us experienced mathematics as a discipline that was relegated to a textbook rather than a discipline that helped us describe and make sense of the world. As a result, many of us still believe that mathematics is a fixed and finite body of knowledge and that doing mathematics means calculating answers to “classic” problems by applying rote techniques.

“...expectations of traditional school practice... demanded that children be told exactly what to do and how to do it ...that the essence of mathematics was conformity to orthodoxy, and that math was no place for thoughtfulness or originality.”
Davis, 1997

“Many parents operate under the myth that “What was good enough for me is good enough for my child.”
NRC, 1989

As adults, we would never be able to survive in retirement on the salaries we earned at the start of our working lives, but we expect our children to survive on the same mathematics curriculum that we experienced at the beginning of our school careers. It is time to seriously reexamine our conceptions of mathematics as a discipline.

This introductory chapter is intended to summarize the changing nature of mathematics and mathematics education. It builds the case for change by synthesizing and summarizing the answers to the questions:

- What is mathematics?
- Why does mathematics education have to change?
- What needs to change?
- How do we change?

What Is Mathematics?

Mathematics is the science of patterns

Physicist Steven Weinberg has described the uncanny ability of mathematicians to anticipate nature (MSEB, 1990, p. 11). They observe, experiment, and make conjectures. They investigate, measure, calculate, and classify to explore their questions. Mathematics is not a passive pursuit nor a spectator sport, but rather an active attempt to identify, describe, and explain patterns wherever they exist in whatever form. As such, mathematics needs to be taught more often as a laboratory experience, with students asking questions, exploring relationships, explaining their ‘theories,’ and defending their positions and strategies.

Mathematics is a universal language

Mathematics is a language for communicating the order in the world. With the advent of computer technology, it has also become the language of manufacturing, finance, social policy, science, and the technical workforce. Mathematics has its vocabulary, its symbols, its definitions, and expressions which help us internalize and clarify our thinking and communicate our ideas. Learning the language of mathematics requires immersion in meaningful activity and conversation.

Mathematics is an art

Mathematics has a long, fascinating, and *continuing* history. Over 200,000 new mathematical theorems are proven each year. In the 1990s alone, more theorems were proven than in all of the years prior to that (Benson & Vessey, 1997, p. 5). The potential for mathematical discovery—the “aha” experience—is in all of us. In fact, with the rapid proliferation of mathematical applications in the workplace and the creative arts, students will need many opportunities to be thoughtful and original as they experience mathematics instruction.

Mathematics is a tool to solve problems

Artists paint, musicians compose, scientists experiment, athletes compete, mechanics repair—and mathematicians analyze and solve problems. Problem solving is at the heart of mathematics, and its component skills—exploring, representing, predicting, testing, generalizing, justifying, communicating, and applying—are learned by doing. Mathematics content and instruction must significantly increase students’ willingness, perseverance, and ability to pursue and solve problems. These important *basics* of mathematics cannot be memorized.

“As biology is a science of living organisms and physics is a science of matter and energy, so mathematics is a science of patterns.”

MSEB, 1990

“Art and music students at all levels have the opportunity to be creative. Mathematics students should have that same opportunity.”

Treisman in Cohen, 1995

Why Does Mathematics Education Have to Change?

There are several good reasons for teaching and learning mathematics well (MSEB, 1990):

- to help individuals solve problems of everyday life
- to prepare students for jobs, vocations, or professions
- to enable citizens to participate intelligently in civic affairs
- to impart a major element of human culture

Compelling evidence indicates that the current system of mathematics education is not accomplishing these goals for most students.

• Student Performance

On national tests of achievement, including the Scholastic Aptitude Test (SAT), the American College Test (ACT), and the National Assessment of Educational Progress (NAEP), Minnesota students generally score near the top when compared to students from other states. The SAT and ACT results generally measure the performance of college-intending students, while the NAEP is the only nationally representative and continuing assessment of what students in the United States and in Minnesota know and can do. The 1996 NAEP results indicate that, while the performance of our fourth and eighth graders places Minnesota in the top group of states, only one in every three students in both grades has mastered the skills and concepts expected at their grade levels. (See the 1996 NAEP summary charts in Appendix D.)

Being among the best in the U.S., however, is not the same as being “first in the world.”

Minnesota’s consistently strong performance in mathematics compared to the rest of the nation looks different in an international context—a major concern for a state whose primary economic competitors are other countries, not other states. The Third International Mathematics and Science Study (TIMSS) compared the United States with forty other nations in curriculum, teaching, and student performance at the fourth, eighth, and twelfth grade levels. The results provide us with some significant opportunities to reflect on our assumptions and improve our practices:

- The United States was the only country whose students dropped from above average performance in mathematics at the fourth grade to below average performance in mathematics at the eighth grade. The same trend is true for Minnesota students, but to a lesser extent.
- The TIMSS data reveal that the content of Minnesota eighth grade mathematics courses is less rigorous than that in most other countries. The present eighth grade curriculum continues to focus largely on number, while students in other countries are studying algebra and geometry.
- Scores on the 4th grade TIMSS data indicate that Minnesota students score no better than United States students as a whole.

The performance of Minnesota students on the mathematics portion of the state’s Basic Standards Test reinforces these concerns. In 1997, only 70% of the almost 52,000 public school eighth graders who took the test passed it at the 75% or better level. Individual district passing rates ranged from 14% to 100% of their students (MDCFL, 1997).

The data is in. Clearly the mathematics achievement of Minnesota students can improve.

• Equity Concerns

Being a state at the “top” in national comparisons does not mean that all Minnesota students are performing at those same levels. Minnesota’s high overall student achievement on the mathematics portion of NAEP masks a poor comparative performance at the fourth and eighth grades when the

“TIMSS is not an answer book, but a mirror through which we can see our own education system in international perspective.”

NCES, 1996

results are disaggregated by race/ethnicity. In fact, among the most alarming statistics is the fact that the average score for Minnesota African American students' at the 4th grade is the *third lowest in the nation* (Reese et al., 1997).

An analysis (Myers, 1997) of the 1996 Minnesota Basic Standards Test data shows that there are significant achievement gaps *statewide* for African American, American Indian, Hispanic, and Asian students. The range of 1997 scores on the same test continue to show a similar pattern, one that is deeply troubling.

We know from ongoing analyses of various sources of data, including SAT and ACT scores and degree completion rates, that these gender and race/ethnicity achievement gaps in Minnesota are persistent and have not significantly improved in the past decade (Johnson, 1996). SciMath^{MN} is exploring the dynamics of these inequities by conducting focus groups around the state, listening to parents', teachers', and students' thoughts on the quality of mathematics and science education in Minnesota (McKendall & Kroll, in press).

Like Garrison Keillor's mythical community of Lake Wobegon, we Minnesotans may take comfort in the wishful belief that "all our children are above average" especially compared to the rest of the United States, but the truth is that while some Minnesota students are above average and some average, too many of our children are performing far below average. If current inequities in our mathematics and science education programs are not addressed directly and systemically, the gap between the educational "haves" and "have-nots" in both our state and our nation will increase.

• Research on Learning

The work of cognitive psychologists provides provocative evidence about how the brain works and how we learn new information. All of us, children and adults alike, are more successful learners if:

- new information is connected to what we already know
- learning is active and engaging
- we are motivated to make sense of fundamental concepts
- we are allowed and even encouraged to interact and converse with others in productive ways

According to Minnesota eighth graders in the survey portion of the TIMSS study, the most frequent instructional activities in the classroom are teacher demonstrations of problems and worksheets (SciMath^{MN}, 1997, p 7). Clearly, the majority of these students' classroom activities do not reflect learnings from cognitive science and brain research. Mathematics must be taught in a more constructive, active way.

• Technological Innovations

In the past century our society's economic system has shifted from a dominance on agriculture to an emphasis on industrial manufacturing to an increasing reliance on technology. Two-thirds of Minnesota's employers believe today's workers need a strong background in technology to succeed (Minnesota Business Partnership, 1993). However, in their advertising, IBM suggests that every industry in America has been revolutionized by new practices and technology with one exception—education (See *Business Week*, October 17, 1994).

Computers and calculators have also profoundly changed the world of mathematics. It is now possible to perform all the techniques of mathematics covered in a K-14 curriculum on hand-held calculators. Just as word processors do not replace good grammar and coherent sentences, calculators and computers do not replace the need to use "mental math." They do provide, however, a

"More than ever before, Americans need to think for a living; more than ever before, they need to think mathematically."
NRC, 1989

dynamic numerical and graphical environment for investigating mathematics problems and statistical applications. As a result, technology has made some mathematics more important and more immediately available to students. The content and structure of the mathematics curriculum need to change to better prepare students with both the mathematics and the technical skills they will need to survive in the “information age” in which computers and calculators are used routinely.

• The Needs of the Workplace

“...almost 90 percent of new jobs require more than a high school level of literacy and math skills.”

Riley, 1998

You don’t have to go far to understand how rapidly the need for mathematics in the workplace is increasing. Visit a farm, tour a bank, travel with a salesperson—the explosive growth of technology has made mathematics essential to more occupations than ever before. In the job market, workers who have strong mathematics and science backgrounds, even if they have not gone on to college, are more likely to be employed and generally earn more than workers with less preparation (U.S. Department of Education, 1997).

Computers have literally pushed the “quantitative envelope” for most vocations. Many jobs that once required little background in mathematics now call for specific skills in algebra, geometry, measurement, probability, and statistics. According to an industry-wide standard, an entry level automobile worker needs to use and apply formulas from algebra and physics to properly wire the electrical circuits of any car (U.S. Department of Education, 1997). As a result, one very important school-to-work connection is the need for students to study more mathematics and to better understand when and how mathematics can be “put to work.”

• Democratic Citizenship

“In our technically-oriented society, ‘innumeracy’ has replaced illiteracy as our principal education gap.”

Jaffe in Romberg, 1992

Read a newspaper, watch coverage of an election campaign, or listen to a public policy debate—graphical, statistical, and financial information is used extensively to influence public opinion. The public, raised on a diet of “one right answer” problems, tends to trust any expert opinion that relies on mathematics. Young citizens will need experience with and skills in critically analyzing and reflecting on the use of mathematical models within the context of social problems. There is no question that quantitative literacy is more important than ever before to make sense of issues in a complex, information society.

In summary, our technological world defines success differently than have our schools (Kansky, p. 44):

“Individual readiness for work is key to industry’s ability to compete.”

Steinwall, 1997

The world emphasizes...	while schools have emphasized...
• making decisions	• performing routine tasks
• solving problems	• memorizing
• finding the BEST solution	• finding THE solution
• working on a team	• working alone
• using appropriate technology	• using paper and pencil
• creating new skills	• practicing traditional skills
• using education to improve employees knowledge and skills	• using education to filter students out of advanced courses

The economy needs academically prepared workers; our young people need jobs. It is time for schools to change.

What Needs to Change?

- Our view of mathematics and who can learn it
- Our expectations regarding what all students must know and be able to do
- How we teach what all students must know and be able to do
- How we measure what all students must know and be able to do
- The quality and quantity of support for teachers and students of mathematics

(Lindquist, 1989; Kansky, 1994)

Our view of mathematics

No employer will ever ask an employee to solve thirty multiple choice questions or give them an answer key when they finish with a task. Mathematics is a dynamic field of knowledge with an almost limitless number of applications, not a limited set of rules to be memorized and practiced. A mutual goal of the national voluntary standards for mathematics education (NCTM, 1989), the *Minnesota Graduation Standards*, and this *Framework* is that students will view mathematics as an interesting and useful subject that makes sense and that they can use confidently to solve relevant problems.

... and who can learn it.

For too long, the ability to understand mathematics has been viewed as a product of inheritance—students either had the “math gene” or they didn’t. Too often teachers used timed tests and quick recall exams to sort those students who had it from those who did not. Changing our view of mathematics challenges our previous assumptions about who can learn mathematics:

- TIMSS analyses reveal that tracking students into ability groups in grades 4-5-6 is a uniquely American phenomenon; in other TIMSS countries, including those that outperform us, all students are heterogeneously grouped through eighth grade.
- Evaluations of newly developed integrated curricula projects indicate that a much higher proportion of students learn more high school mathematics because of increased participation and greater understanding.
- In many districts and states that have raised the achievement bar, the passing rates have tended to remain constant, or, in some cases, improve (Barth, 1997, p. 33). For example, the passing rate of Hispanic students on the Texas state mathematics test has improved from 47% to 72%. “By shining a light on various subgroups at the same time it is raising standards, Texas has managed to pursue both excellence and equity” (Olson & Hendrie, 1998, p. 36).
- Finally, Minnesota eighth graders scored at about the world average in mathematics, but in science essentially the same group of students scored near the top of the 41 participating countries. These results show that the critical difference in achievement lies not in who’s taking the test, but in the fundamentals of education—what’s taught, how it’s taught, and how it’s tested (SciMath^{MN}, 1997).

The data cannot be ignored: challenging mathematics education can build real opportunities for students who might not otherwise have them. In fact, taking challenging mathematics and science courses is a more important factor in determining college attendance than is either a student’s family background or income (U.S. Department of Education, 1997). “Math for all” is more than a slogan; it is an issue of social justice and a national economic necessity.

“There is no educational justification for keeping so many young people out of higher level courses.”

Barth, 1997

“It is no longer possible to earn high wages with low skills. And it is no longer possible to function effectively in an increasingly complex society without the intellectual tools to navigate that complexity. All students need the capabilities that today only a small minority can reach.”

Rothman, 1996

Our expectations regarding what all students must know and be able to do

The Third International Mathematics and Science Study (TIMSS) examined and compared curriculum outlines, frameworks, textbooks, and teaching practices. The findings indicate that our vision in the United States for what mathematics should be taught and how it should be taught is *splintered*, producing unfocused curricula and 400-page textbooks that attempt to cover a little bit of everything. The amount of repetition from year to year guarantees that students will practice the same mathematics over and over again (whether or not they understood it the first time) without ever delving more deeply into the concepts.

“To the Romans a curriculum was a rutted course that guided the path of two-wheeled chariots. Today’s mathematics curriculum—a course of study—follows a deeply rutted path directed more by events of the past than by the changing needs of the present.”

MSEB, 1990

To reach the *Minnesota Graduation Standards* in mathematics, all students will have to engage in a more challenging, focused, and coherent curriculum than that which presently exists for the majority of students. Such a curriculum must be organized around the “big ideas” in mathematics—the major concepts in the discipline—and developed in such a way that it builds a progressively deeper level of student understanding while avoiding needless repetition.

Elementary school mathematics must provide an effective foundation for mathematics concepts, integrating number sense understandings and skills in subject areas beyond arithmetic, including geometry, measurement, data investigation, and probability. Understanding of basic facts and whole number calculation should precede their memorization, and application of whole number computation should occur in context as often as possible.

Middle school mathematics must move beyond arithmetic to help students develop increasingly complex problem solving skills. The broadened concept of the basics must include proportional reasoning skills and algebraic and geometric thinking, with mathematics content related to everyday situations as well as other school subjects.

High school mathematics must prepare students for all that comes later: the workplace, college, and citizenship. It must reflect a common core of broadly used mathematics including algebra, geometry, data analysis, probability, and discrete mathematics, with an emphasis on developing fluency with symbols and an appreciation for abstraction and formal mathematical reasoning (MSEB, 1991, pp. 10-11).

Minnesota is a state renowned for its local control. Achieving a coherent vision for what mathematics our students should know and be able to do will require a broad conversation and consensus building process. While this *K-12 Mathematics Framework* cannot guarantee student achievement, it can provide the focus for those conversations as well as a central guiding vision.

How we teach what all students must know and be able to do

What happens in our classrooms is a reflection of the lack of focus in our curriculum. Well-meaning teachers do what the textbooks and public expect: they attempt to teach everything, and end up teaching few topics well. Because there is no time to teach any topic in much depth, teachers merely skim the surface, never moving beyond procedural application of facts and algorithms. A videotape study of eighth grade classroom instruction in Germany, Japan, and the U.S. indicates that American students rarely study important mathematics in thoughtful ways. In reality, classroom mathematics lessons in the United States resemble television programs with many commercial breaks, rather than a smooth-flowing story with an identifiable beginning, middle, and end.

Most teachers believe they have the capacity to be more effective in the classroom. With time and professional support, we know the following characteristics of teachers’ instructional practice can go a long way to increasing student achievement. Teachers can and must:

“Teaching...is more like an agricultural process. You help to grow ideas in other people’s minds. You need water, good soil, sunshine, fertilizer ...but the thoughts that grow there are their own.”

Ahlgren in Hardy, 1998

- make mathematics instruction more *conceptual*—emphasize understanding of fundamental concepts over recall of algorithms
- use *concrete* materials, pictures, and diagrams to help students explore and understand mathematical concepts and relationships
- help students build *connections* from new learnings to a) their prior knowledge; b) previous lessons; c) their cultural backgrounds; d) real life; e) other areas of mathematics; f) other subjects
- structure a learning environment in which *collaborative* problem solving and mathematical *conversations* are the norm
- recognize that students ultimately have to make sense of the mathematics in order for real learning to occur; they need time and opportunities to struggle with new mathematical ideas until they can fit them into their mental framework

Teaching is not simply making broadcasts or presentations, nor can it be defined by recipes or “how to” lists. Different students learn in different ways and good teachers are role models who plan, question, moderate, experiment, design, and choreograph in order to maximize student learning.

How we measure what all students must know and be able to do

Student assessment is a process of gathering evidence, interpreting that evidence, and using the results to improve student learning and instruction. This suggests that assessment practices must be systematic, not one-shot or hit-or-miss activities. For this work, teachers of mathematics need to consider evidence from multiple sources, including written tests or problem solutions, interviews, observations, projects, presentations, and work over time organized in portfolios. Different assessments are needed for different purposes and the process itself is continual and recursive. Assessing for conceptual understanding and for students’ mathematical power is not as easy or direct as counting the number of basic facts correct out of 100 exercises. Teachers will need support as they explore new assessment tools and strategies and make inferences from more open-ended student work.

The primary goal of assessment is to improve student learning and advise modifications in instruction. The following ABC’s of assessment should guide our work:

Characteristic:	Ask:
a - authentic	Is this how people use this information or these skills in the real world?
b - unbiased	Do all my students have an opportunity to be successful doing this task?
c - constructivist	Are students making the decisions and doing the thinking in this work, or are they simply following my directions?
d - developmentally appropriate	Does this work reflect each student’s intellectual, physical, and/or psychological development?
e - embedded	Is this task part of the natural flow of classroom activities?
f - focused	Does this task reflect the content we have studied as well as my instruction? Do my content, instruction, and assessment tasks ask my students to demonstrate the learning required in the standard?

g - generalizable	If my students can do these tasks, can I be reasonably sure they possess the knowledge and skills called for in the whole standard?
h - high in rigor	Does this task require my students to think and demonstrate a challenging level of knowledge and skill?
i - interesting	Will this task engage my students and me?

The quality and quantity of support for teachers and students of mathematics

"A nation that can put a man on the moon ought to be able to reform its educational system—so goes the standard claim. But changing how and what mathematics is taught to our children is not a technical problem. It is a human problem that demands an understanding and appreciation of how people work together in classrooms to learn and teach and do mathematics."
Kilpatrick, 1997

• Time

Time should be the variable and learning should be the constant in mathematics education. We need to use time in new and better ways to meet the individual needs of learners as well as provide teachers with the time they need for planning, preparation, collaboration, reflection, and professional growth (National Education Commission on Time and Learning, 1994). Achieving mathematics standards will require the restructuring of time for almost all students and teachers.

• Resources

The majority of U.S. and Minnesota teachers rely on their textbook to determine what mathematics to teach and how to teach it (SciMath^{MN}, 1997; Schmidt, et al., 1996). Textbooks alone, however, are not sufficient for learning and teaching mathematics. Students must have regular access to and opportunities to effectively and appropriately use calculators, computers, and physical materials to do mathematics. It is also crucial to identify and provide effective curriculum materials for teachers to help them:

- understand more rigorous mathematics content
- plan more effective instructional practices, and
- assess student learning in multiple ways

• Professional Development

Teachers are key to improving student learning of mathematics and both inservice and preservice teachers will need considerable support to "teach better mathematics and teach mathematics better" (Riley, 1998). Effective professional development must provide long-term, intensive assistance as teachers work hard to improve their own understanding of mathematics, deliver more effective instruction, assess in meaningful ways, and monitor student achievement. Raising the bar for teachers will take some serious capacity building across the nation as well as a major commitment of time, money, and professional resources from all segments of the state and local educational system. That work needs to start now.

• Incentives for students and teachers

Holding the belief that only those students on the right-hand tail of the bell-shaped curve can be successful in school and in life gives students little incentive to work harder. Grading students by comparing them to one another gives them no additional reason to make a greater effort (Rothman, 1996, p. 2). School can be compared to gymnastics: students need clear expectations of what a "10" looks like and what the rewards are for earning it if they are to put forth the effort that will enable them to learn and succeed.

“If you ask teachers to change the way they deal with students and to relate to their colleagues differently, the incentives that operate at the organizational level have to reinforce and promote those behaviors.”

Elmore, 1996

“In every country that delivers a ‘world-class’ education, communities and parents exhibit a high regard for education, a healthy respect for teachers, and a holy regard for learning—all key ingredients in an ‘education ethic’ that creates positive expectations for student learning.”

Haynes & Chalker, 1997

“Looking at student work is an amazingly economical way to promote understanding of what is being taught and learned.”

Mitchell, 1996

Changing teacher practice even for committed teachers takes a long time and several cycles of trial and error (Elmore, 1996, p. 24). The most compelling incentives for the majority of teachers is evidence of increased student learning, along with opportunities to network and collaborate professionally. School and district personnel need to consciously address the conditions under which teachers work to create structures that promote new instructional practices. These leaders must motivate and recruit the best and the brightest into teaching, plan for incentives that will compel teachers to change their practices, and allow teachers regular opportunities to influence and reinforce each other’s work.

• Public Support

U.S. Secretary of Education Richard W. Riley has emphasized the need to focus on the students and the interests of the nation and identify our common ground. “We all want students to master the traditional basics—to be able to add, subtract, multiply, and divide, and be accurate and comfortable with simple mental and paper and pencil computation. We all want students to have the opportunity to master challenging mathematics, which for K-12 students includes arithmetic and algebra, geometry, probability, statistics, trigonometry and calculus. We also want students to master the basics of the information age—problem solving, communicating mathematical concepts, and applying mathematics in real-world settings” (1998). Public support for this kind of change is critical for success. Parents, business leaders, legislators, and other community members all need to understand the stake they have in raising student achievement and do their part to help teachers and students make it happen.

How Do We Change?

First and foremost, we need to shift from a system that sorts students by some measure of innate ability to one that expects and enables all students to do well. We must believe that the capacity to use one’s mathematical abilities in the world is learnable, and that effort, not native ability, results in mathematical achievement.

Second, we need to set clear expectations for what students must know and be able to do in mathematics, teach to those standards, assess student work against those standards, and celebrate achievement of those standards. These standards for student performance must be clear, explicit, and well understood by students, parents, and teachers at all times (Rothman, 1996, p. 4). With clear statements about student expectations for learning, teachers can focus their professional conversations on student work and focus their professional energies on helping students achieve. With clear expectations for performance, students have a visible target at which to aim their efforts, and teachers, students, and parents can better gauge the additional support students may need to be successful.

Third, we need to develop benchmarks for mathematics achievement at developmental anchor points. Factories no longer wait until the end of the assembly line to identify “defects”—standards for quality work are applied at checkpoints along the way. Likewise, teachers, students, and parents need opportunities to adjust their approaches, increase their effort, or provide for enrichment before a student completes his/her K-12 mathematics education.

Fourth, we must expend time and energy to organize and facilitate groups of teachers (and administrators, parents, school board members, etc.) to look at student work and compare it to the standards. Critically examining student work is a crucial element of professional development that focuses everyone’s attention on student learning. (What concepts do these students know, and what mathematics can they do? Was this task designed appropriately to help students learn impor-

tant mathematics? What does this student work tell us about how students learn?) Looking at student work is the vehicle for internalizing standards, for making them part of the culture of teaching and learning mathematics, so all stakeholders can work together to help students attain them.

Finally, it is imperative that our work of reforming mathematics education does not itself become “a mile wide and an inch deep.” We must emphasize in everything we do the careful alignment of curriculum, instruction, and assessment. We must always focus our efforts on that work that will accomplish the most for student learning. When we get all stakeholders focused on the goal of increasing student achievement in mathematics, then we will have a chance to make a difference.

Summary

Children today will experience a world very different from the one in which their parents were raised. Technological innovations and the reality of a global economy have changed the way we live and work. Mathematics will play a crucial role in preparing our children to take on their roles as workers, citizens, and parents in the twenty-first century.

To succeed personally and professionally, all children will need more mathematics—

- relevant mathematics that makes sense in their world
- rigorous mathematics that gives them a deep understanding and facility with fundamental concepts
- rewarding mathematics that motivates them to continue studying it throughout their lifetimes

The basic skills for all our children have changed. While students will continue to need facility in computation and “mental math,” they need much more. They need the habits of mind, the problem solving skills, and the technological “know-how” that are developed by:

- contextual, coherent, and connected curricula
- instructional strategies informed by best practice research
- multiple tools that help teachers assess what students know and can do

These are the characteristics of standards-based instruction. Standards-based mathematics is mathematics that measures up to the challenges of educating our children today for their futures tomorrow.

Mathematics education can and is changing to address this reality. With changes grounded in the *Minnesota Graduation Standards*, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), and this *Framework*, students will be better equipped to go beyond traditional mathematics. Action on these recommendations will help us move all students off the mathematical “sidelines” and get them into the game of *doing* mathematics.

“[We] must restructure the mathematics curriculum—both what is taught and the way it is taught—if our children are to develop the mathematical knowledge (and the confidence to use that knowledge) that they will need to be personally and professionally competent in the twenty-first century.”
MSEB, 1990

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